

## MATH 1D, HW #1 – QUESTIONS

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**Instructions:** Choose **three** questions out of the **four** below to complete! If you attempt all 4, your three highest scores will be recorded as your grade. Also, write me an email if you get stuck! These are somewhat complex questions.

**Question 0.1.** Prove that

$$\lim_{n \rightarrow \infty} \frac{\alpha(n)}{n} = 0,$$

where  $\alpha(n)$  denotes the number of primes that divide  $n$ .

**Question 0.2.** Prove that every convergent sequence is bounded.

**Question 0.3.** Prove that the sequence

$$\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}, \dots$$

converges. What does it converge to?

**Question 0.4.** Consider the sequence

$$\{\alpha_n\}_{n=1}^{\infty} = \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \dots$$

For which numbers  $\beta \in \mathbb{R}$  is there a subsequence of  $\{\alpha_n\}_{n=1}^{\infty}$  that converges to  $\beta$ ? Find all such values of  $\beta$ , and prove that your answer is correct.