MATH 1D, MIDTERM – QUESTIONS

INSTRUCTOR: PADRAIC BARTLETT

Instructions: Do four of the following six questions. If you do more than that, your top four scores will be graded. Good luck! You have until Jan. 28th at 4; as a reminder, the test is open notes/books and collaboration is not allowed.

Question 0.1. Prove that the following limits hold:

$$(A) \qquad \lim_{n\to\infty} \frac{n}{n+1} - \frac{n+1}{n} = 0.$$

$$(B) \qquad \forall r > 0, \lim_{n\to\infty} \sqrt[n]{r} = 1.$$

$$(C) \qquad \forall a, b > 0, \lim_{n\to\infty} \sqrt[n]{a^n + b^n} = \max\{a, b\}.$$

Question 0.2. Determine whether the following sums converge or diverge, and prove your answer:

$$(A) \qquad \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$$

$$(B) \qquad \sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$$

$$(C) \qquad \sum_{n=2}^{\infty} \frac{n^2 \log(n) \cdot 2^n}{n!}$$

Question 0.3. Prove that a sequence cannot converge to two different limits.

Question 0.4. Prove that if the sum $\sum_{n=1}^{\infty} a_n^2$ converges, then so must the sum $\sum_{n=1}^{\infty} \frac{a_n}{n^\beta}$, for any value of $\beta > 1/2$. (hint: look at the second HW!)

Question 0.5. Suppose that the sum $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, and $\{a_{n_k}\}_{k=1}^{\infty}$ is a subsequence of $\{a_n\}_{n=1}^{\infty}$. Show that $\sum_{k=1}^{\infty} a_{n_k}$ is also absolutely convergent.

Question 0.6. Find a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that

- $\lim_{x\to\infty} f(x)$ does not exist, and
- $\int_0^\infty f(x)dx = 1$.

Date: Due Date: Thursday, Jan. 28, at 4 p.m.