Math 1c TA:

Homework 4: More Applications of the Derivative in $\mathbb{R}^n$

Week 4 *Caltech 2013*

For details on the collaboration policy, due dates, etc., please refer to the Ma1c course webpage. If you have any questions when working on the HW, please don't hesitate to contact your TA (or really any of the TA's,) or indeed even your fellow students!

As always, show your work.

#3.4.6. Find the extrema of the function $f(x, y, z) = x + y + z$, given the two simultaneous constraints $x^2 - y^2 = 1, 2x + z = 1$.

#4.2.2. Find the arc length of the curve $\gamma(t) = (1, 3t^2, t^3)$ over the interval $0 \leq t \leq 1$.

#4.2.5. Find the arc length of the curve $\gamma(t) = (t, t, t^2)$ over the interval $1 \leq t \leq 2$.

#4.3.11. Sketch a handful of flow lines (i.e. five, or as many as it takes for you to have a good picture of what's going on) for the vector field $\mathbf{F}(x, y) = (y, -x)$.

#4.3.16. Show that the curve $\mathbf{c}(t) = (t^2, 2t - 1, \sqrt{t})$ is a flow line of the velocity vector field $\mathbf{F}(x, y, z) = \left(y + 1, 2, \frac{1}{2z}\right)$.

#4.3.18. Show that the curve $\mathbf{c}(t) = \left(\frac{1}{t^3}, e^t, \frac{1}{t}\right)$ is a flow line of the velocity vector field $\mathbf{F}(x, y, z) = (-3z^4, y, -z^2)$

#4.4.1. Find the divergence of the vector field $\mathbf{V}(x, y, z) = e^{xy}\mathbf{i} - e^{xy}\mathbf{j} + e^{yz}\mathbf{k}$.

#4.4.13. Find the curl $\nabla \times \mathbf{F}$ of the vector field $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

#4.4.27. Suppose that the three functions $f, g, h : \mathbb{R}^2 \to \mathbb{R}$ are all differentiable. Prove that the vector field $\mathbf{F}(x, y, z) = (f(y, z), g(x, z), h(x, y))$ has zero divergence.

#4.4.34. Show that the vector field $\mathbf{F}(x, y) = (x^2 + y^2)\mathbf{i} - 2xy\mathbf{j}$ is *not* a gradient field.