Math 1c TA: Padraic Bartlett

Homework 1: Open and Closed Sets; Limits and Continuity

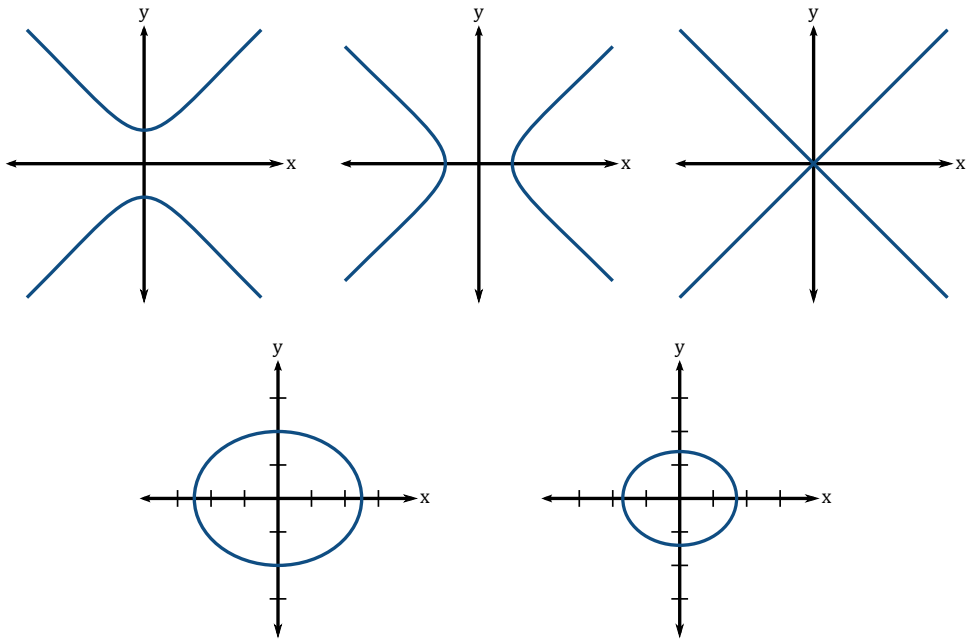
Week 1 *Caltech 2013*

A number of students have emailed asking if there are differences between the fifth and sixth editions of the textbooks and/or if we could post the transcribed problems for each week's HW, instead of just the problem numbers. Accordingly, we're transcribing the HW! Democracy: it works.

For details on the collaboration policy, due dates, etc., please refer to the Ma1c course webpage. If you have any questions when working on the HW, please don't hesitate to contact your TA (or really any of the TA's,) or indeed even your fellow students!

#2.1.3. Five level curves are given below, along with five corresponding graphs. Match up the given level curves to their graphs. (No explanation is necessary, but you're welcome to provide it anyways.)

- $x^2 - y^2 = 1$.
- $x^2 - y^2 = 0$.
- $x^2 - y^2 = -1$.
- $2x^2 + 3y^2 = 6$.
- $2x^2 + 3y^2 = 12$.



#2.1.9. Let S be the surface in $\mathbb{R}^3$ defined by the equation $x^2y^6 - 2z = 3$.

(a) Find a real-valued function $f(x, y, z)$ and a constant c such that S is the level set of f at value c.

(b) Find a real-valued function $g(x, y)$ of two variables such that S is the graph of g.

#2.1.27. Sketch the surface corresponding to the equation $4x^2 + y^2 = 16$.

#2.2.4. Find the following limits, or show that they do not exist. Prove your claims.

(a) $\lim_{(x,y)\to(0,1)} e^x \cdot y$.

(b) $\lim_{x\to 0} \dfrac{\sin^2(x)}{x}$.

(c) $\lim_{x\to 0} \dfrac{\sin^2(x)}{x^2}$.

#2.2.10. Find the following limits, or show that they do not exist. Prove your claims.

(a) $\lim_{(x,y)\to(0,0)} \dfrac{e^{xy}}{x+1}$.

(b) $\lim_{(x,y)\to(0,0)} \dfrac{\cos(x) - 1 - (x^2/2)}{x^4 + y^4}$.

(c) $\lim_{(x,y)\to(0,0)} \dfrac{(x-y)^2}{x^2 + y^2}$.