Math 1c	TA: Padraic Bartlett

Recitation 10: Final Review - Sample Questions

Week 10 *Caltech 2012*

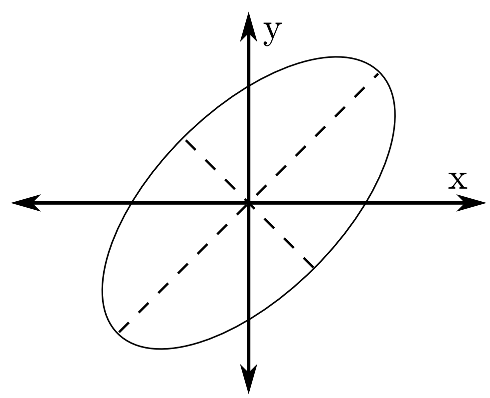
The following six questions are intended to simulate a possible final exam, and to test your mastery of the material covered in the second half of Ma1c. These problems are mostly somewhat harder than the problems you will encounter on the real test; in theory, if you're comfortable dealing with problems like those below, you will do well on the final!

Solutions will be posted ideally sometime on Saturday, so that you can check your work there.

1. Consider the ellipse

$$\left(\frac{x-y}{2}\right)^2 + \left(\frac{x+y}{4}\right)^2 = 1,$$

depicted below.

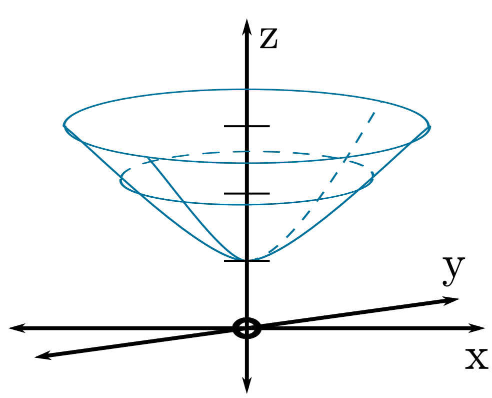


What is the average value of the function $F(x,y) = x^2$ on the interior of this ellipse?

2. Suppose that S is the upper sheet of the hyperboloid with two sheets

$$z^2 - x^2 - y^2 = 1, z \in [1,3],$$

depicted below:



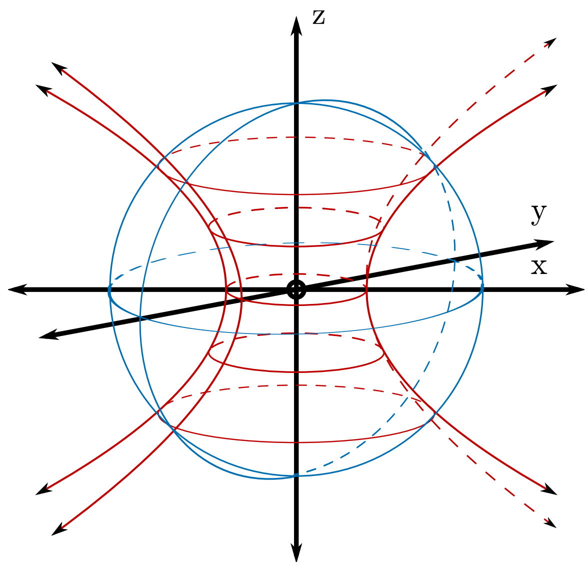
Let $F(x, y, z) = (-x, -y, -z)$ denote a vector field modelling a snowfall: i.e. at any point (x, y, z) in $\mathbb{R}^3$, F indicates the magnitude of snow flowing through $(-x, -y, -z)$, along with the direction it flows in.

Assume that F is denoting inches of snow accumulated per hour. How much snow accumulates on our surface S in a hour: i.e. what is the total flow $\iint\limits_S F \cdot ds$?

3. Let S be the surface given by taking the portion of the hyperboloid of one sheet

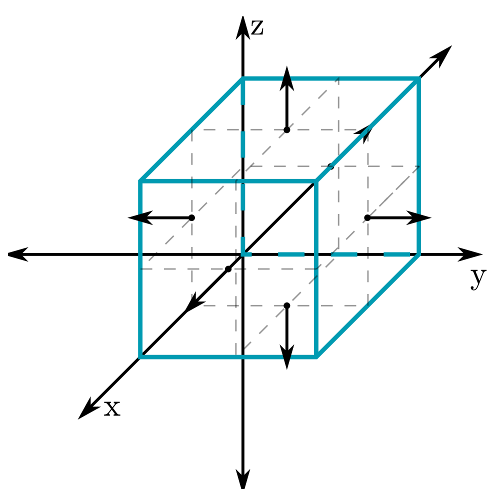
$$H_1 = \{(x, y, z) : x^2 + y^2 - z^2 = 1\}$$

contained by the sphere of radius 4, as depicted below:

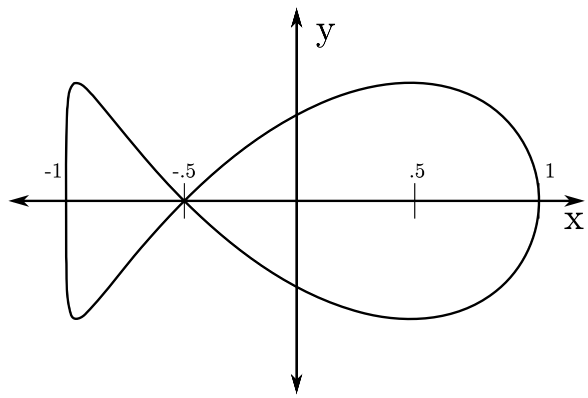


Set up (but don't calculate) an iterated integral for the surface area of S.

4. Directly calculate the integral of $F(x, y, z) = (3x^2y, -3xy^2, z)$ over the surface of the unit cube with the outward orientation shown in the picture below. Then, use the divergence theorem to calculate this in a much faster manner.

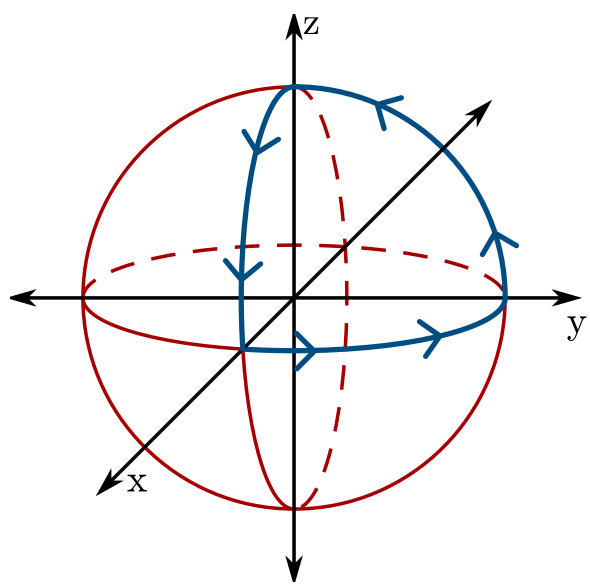


5. Let $c(t) = \left(\cos(t) - \frac{\sin^2(t)}{2}, \cos(t)\sin(t)\right)$ denote the "fish curve" drawn below:



Find the area contained within this curve.

6. Let $S = \{(x, y, z) : x^2 + y^2 + z^2 = 1, x, y, z \geq 0\}$ and $C^+ = \partial S$ be the boundary of S traversed in the counterclockwise direction from high above the z-axis, as depicted below:



Let $F(x, y, z) = (x^4, y^4, z^4)$ be a vector field. Calculate $\int_{C^+} F \cdot dc$ directly, then use Stokes's theorem to calculate it with much less effort.