# Final Exam Study Guide Math 34A

*All practice problems were written to be doable WITHOUT a calculator.*

*Reminders about the test*

- The final will be during class on Thursday, July 29
- Bring a photo ID to the exam
- There are NO calculators
- There are NO books or notes allowed for the exam
- You are allowed a 3 $\times$ 5 notecard with whatever you want written on both sides

*How to show your work so the grader can follow easily + other tips:*

- Include all non-obvious steps. If you are ever unsure whether to include a step, write it down to be safe.
- For word problems, declare any variables (or appropriately label a diagram) before using them.
- Simplify your answers (combine like terms, finish simple arithmetic, simplify fractions).
- Box your answers.
- Double-check that your answer makes sense (this is particularly important with word problems).
- Don't forget to include units in your final answer, when applicable, and be sure those units make sense.
- If you make an error, erase it or cross it out, so the grader does not mistake it for work you want to be graded.
- If the arithmetic gets too crazy, you are probably doing something wrong.
- If you are in need of more practice problems, check out the problems in the textbook or the practice tests in the back.
- Don't cheat.

*Chapter 7*

1. Exponent rules
2. Base 10 log definition
3. Log rules
4. Finding logs and antilogs with a log table
5. Reading the $10^x$ graph
6. Solving algebraic problems using logs
7. Application of logs: Bank interest type problems
8. Application of logs: Half-life and doubling time problems
9. Application of logs: Exponential growth and decay using other bases
10. Logs to other bases (in particular: the natural log)

*Practice Problem:*
Given that $\log(7) = 0.8451$ and $\log(2) = 0.3010$, calculate the following:
(a) log(28)
(b) log(0.0049)
(c) antilog(3.3010)
(d) antilog(-1.1549)

*Practice Problem:*
Suppose a colony of bacteria is growing exponentially. Currently the colony is 6 times as large as it was 5 days ago. What is the doubling time (you may leave logs in your answer)?

*Practice Problem:*
Solve for $t$:

$A \times B^t = \frac{C}{D^{t-2}}$

*Practice Problem:*
Solve for $x$:

$2log(x + 2) = log(x + 2) + 1$

Be careful that your solutions for $x$ don't make any of the above logs undefined!

*Practice Problem:*
Andy invests \$100 in an account that gains interest at a rate of 3% a year. Bob invests \$50 in an account that gains interest at a rate of 2% every *half* a year. How many years would it take until Bob's account has the same amount of money as Andy?

*Practice Problem:*
A certain radioactive isotope decays exponentially with half-life 84 years. If we start with 32 grams of the isotope, how many years will it take until there are 7 grams left? You may leave logs in your answer.

*Practice Problem:*
Suppose that a colony of 20 rabbits in 1990 increased to a population of 150 in 2002. Assuming the population of rabbits is increasing exponentially, how many rabbits will there be in 2010?

*Practice Problem:*
Solve for $x$ using natural logs:

$$10^{x+4} = 8e^{3x}$$

*Chapter 8*

1. Average rate of change
2. Limit definition of the derivative
3. What the derivative means
4. Differentiation rules
5. Tangent line approximation
6. The second derivative and what it means
7. Finding where a function is increasing/decreasing and concave up/concave down
8. Finding maxima and minima

*Practice Problem:*
Find the average rate of change of $f(x) = 5x^2 - 3$ between $x = 2$ and $x = 3$. Now find the average rate of change between $x = 2$ and $x = 2 + h$. What is the instantaneous rate of change at $x = 2$?

*Practice Problem:*
Using the limit definition of the derivative, find $f'(x)$ when $f(x) = 3x^2 - 2$.

*Practice Problem:*
Suppose an object is thrown off the top of a building and its height in meters after $t$ seconds is given by the function $f(t) = 50 + 5t - t^2$. How fast is the object traveling when it hits the ground? When does the object attain its maximum height?

*Practice Problem:*
Evaluate the following derivatives:

(a) $\frac{d}{dx}(4e^{x/2})$

(b) $\frac{d}{dx}(\frac{x^2-3}{x})$

(c) $\frac{d}{dx}(\frac{2}{\sqrt[5]{x}})$

(d) $\frac{d}{dx}(\frac{x^2-4}{x+2})$

*Practice Problem:*
Let $f(x) = \frac{1}{x}$. Find an equation for the tangent line to $f(x)$ at $x = 2$. Then use this tangent line to approximate $\frac{1}{3}$. What is the resulting percent error?

*Practice Problem:*
Let $f(x) = -x^3 + 6x^2 - 12x - 5$. For what values of $x$ is $f(x)$ decreasing? For what values of $x$ is $f(x)$ concave down?

*Practice Problem:*
Suppose $T(p)$ represents the time it takes in minutes to get a sandwich at the Arbor Subway when there are $p$ people in line. What does $T(3) = 5$ mean? What does $T'(3) = 1$ mean? What does $T^{-1}(8) = 6$ mean?

*Practice Problem:*

Suppose $f(t)$ represents the amount of liters in a water tank after $t$ days. What does $f'(4) = -2$ mean? What does $f''(4) = .5$ mean? If $f''(t)$ is always equal to .5, at what time will the tank no longer be losing water?

*Practice Problem:*

Suppose some kids decide to open a lemonade stand on a street corner. It costs them 20 cents to make a single cup of lemonade. After weeks of selling, they find that they can sell 50 cups when they charge $1 per cup and that their sales fall by 5 for each 10 cents extra that they charge. What price should the kids sell their lemonade to make the most profit? What is the maximum profit they can make?

*Practice Problem:*

Suppose a farmer wants to make a rectangular field that is bordered by a brick wall on one side and wooden fence on the other three sides. The cost of the brick wall is $5 per meter and the wooden fence costs $3 per meter. If the farmer only has $120 to spend, what is the maximum area this field can be?