Before asking your mentors about a problem, make three different attempts to solve it.

Our goal is to sample several different ways of calculating solutions to rather specific (not always linear) types of DE’s. This is to help you realize how vast the study of DE’s is, and how creative you must be to solve the Joe-average DE off the street, if humans can even find an explicit solution. Once someone has found a method for solving some class of DE’s, it often works for only a very specific class of functions. Here are some examples of perhaps more obscure (at least to you and me) classes of DE’s and the methods people have found to solve them. Notice that many methods presented here involve reducing the problem to solving a different DE with nicer properties like linearity or separability.\footnote{These examples come from WeBWork problems or problems from your reader.}

1. \textbf{Solving Euler-Homogeneous DE’s} (From WeBWork): Many DE’s are not separable but can be made separable with the appropriate change of variables. One example is the class of first-order differential equations with right hand sides that are functions of the combination $\frac{y}{t}$ or $\frac{t}{y}$. A differential equation of the form

$$\frac{dy}{dt} = f\left(\frac{y}{t}\right)$$

is called Euler-homogeneous. Given such a DE, let $v = \frac{y}{t}$. By the product rule, from $y = vt$ we obtain

$$\frac{dy}{dt} = v + t \frac{dv}{dt},$$

and you will need to use this information to make a DE in $v$. 

\textsc{SIMS Math 3C Final Project}
Due Thursday August 26th
Summer 2010, Grace Kennedy

NAME: 

SIMS Website: http://www.epsem.ucsb.edu/summer_programs/sims.html
Course Website: http://math.ucsb.edu/~kgracekennedy/SIMSsummer2010.html
Use this change of variables to solve the following IVPs:

(a) 
\[ \frac{dy}{dt} = \frac{y + t}{t}, \quad y(1) = 1, \quad t > 0. \]
(b) \[
\frac{dy}{dt} = \frac{y^2 + t^2}{yt}, \quad y(1) = -2, \quad t > 0
\]
2. **Useful change of variables** (From Reader): Use the substitution $z = \ln(y)$ to find the general solution in the DE

$$\frac{dy}{dt} + ay = by \ln(y)$$

in terms of constants $a$ and $b$. 
3. **Bernoulli equation** (From Reader): The nonlinear equation

\[
\frac{dy}{dt} + p(t)y = q(t)y^\alpha
\]

where \( \alpha \neq 0, \alpha \neq 1 \) is called a Bernoulli equation and can be transformed into a linear equation.

(a) Show that the substitution \( v = y^{1-\alpha} \) gives a linear DE in \( v \).

(b) Use part a to solve the Bernoulli equation

\[
y' - y = y^3.
\]
4. **Switching dependent and independent variables** (From Reader):
We will solve some first-order, non-linear DE’s by reinterpreting the DE with $y$ as the independent variable and $t$ as the dependent variable.

Hint: Use the fact that $\frac{dy}{dt} = 1/\frac{dt}{dy}$, where $y = y(t)$ and $t = t(y)$ are inverse functions.

(a) Using this method, solve the following IVP:

$$\frac{dy}{dt} = \frac{1}{t + y}, y(-1) = 0.$$  

(It is enough to get an implicit solution.)
(b) Using this method, find the general solution to the following DE:

\[
\frac{dy}{dt} = \frac{y^2}{e^y - 2ty}.
\]

(It is enough to get an implicit solution.)