

STUDY GUIDE FOR 3C

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Spring 2009

CONTENTS

1. Introduction	1
1.1. When we can find a solution explicitly.	2
1.2. When you don't or can't find a solution explicitly: Qualitative Analysis of the Solutions.	4
1.3. Applications	4
2. Systems of Differential Equations	5
2.1. Phase Diagrams	5
2.2. Linear Algebra	6
3. Review	7
4. Study tips	7

Please keep in mind that this is up to date as of Thursday May 7th. Anything covered in class or section between now and the midterm is fair game.

If you find any errors in content or formatting or if there is something that I have left off, please send me an email and let me know. I hope this helps!

Do not walk into the exam without a thorough understanding of the following information:

- variation of parameters
- separation of variables
- applications from Section 1.3
- definition of a system of differential equations (corresponding solutions)
- phase diagrams and associated vocabulary
- matrix operations
- material covered in the week before lecture

These topics are *absolutely must know*. To be truly prepared for the exam, you should also be comfortable with all of the material covered in homework problems, lecture notes, and any handouts from section. Variation of parameters and separation of variables are on here because you need to know them to work the application problems. (Review material from 3B also still applies.)

1. INTRODUCTION

I'm sure you have noticed, 3C is all about the differential equations (DE's).

- (1) What is a differential equation?

- (2) What is a solution to a differential equation?
- (3) How do we find solutions to differential equations?
- (4) Can we always find solutions to Differential equations?
- (5) What do we do if we can't find a solution? What kinds of questions can you be asked?

1.1. **When we can find a solution explicitly.** Sometimes, when your differential equation is in a nice form, you can find a function $y(t)$ that is a solution. Here is a list of methods and types of problems you can be asked to solve.

- (1) Solving a DE using Separation of Variables: *How do you know when you can apply this method? Explain in your own words how to do this.*
- (2) Solving an Euler-homogeneous DE: *What is an Euler-homogeneous DE? How do you know when you can apply this method? Explain in your own words how to do this.*
- (3) Solving IVP problems. *What are they and how do you solve them.*
- (4) *Explain the difference between the following questions:*
 - Find all constant solutions.
 - Find all non-constant solutions.
 - Find all unstable equilibrium solutions.

- Find all solutions.
- Find all solutions passing through $y(0) = 3$.
- Is $y(t)$ a solution to the following differential equation?

How do items 1-4 fit together?

Here's a cool question. Given $y'(0) = 1$,

- (1) Give all constant solutions through the origin.
- (2) Are there any trigonometric solutions through the origin? What are they?
- (3) Are there any polynomial solutions through the origin? What are they?
- (4) Do you think it is possible to find all of the solutions through the origin?
- (5) Is this a differential equation?
- (6) Contrast this with the problem from the pop quiz: how many solutions are there to the differential equation $(y')^2 = y^2$.
- (7) What is the exact statement of Picard's Theorem?

1.2. When you don't or can't find a solution explicitly: Qualitative Analysis of the Solutions. Sometimes you are given a differential equation that is difficult (or impossible) for you to solve, but you can still be asked questions about the solution.

- (1) You can pick off information from the sign of the derivative: $y' = ky$ models a population (Malthus's model). Under what conditions will the species become extinct, take over the planet, remain constant (Worksheet 1)?
- (2) Slope fields. What are they? How do you draw them? What can you be asked about them?
- (3) Equilibrium solutions. What are they? What does it mean to be stable, unstable, or semistable? (Also under Section 1.1.)
- (4) Basins of Attraction. What are they?
- (5) Most of these give information about the long term behavior of a differential equation given an initial value of y . Explain.
- (6) Orthogonal trajectories. What are they? How do you find them? How can you check if two sets of curves are in fact orthogonal sets of curves?
- (7) Euler-approximation. What is it? How can you use it to approximate a solution to a DE and why does it work? How can you use it to approximate the number e ?

1.3. Applications. Right after the first midterm, you did a lot of applications. Try writing your own of each of these type problems and answering them. Make sure you understand how to set up each of the differential equations that models what is going on. Don't just plug into a formula.

- The lottery problem- How long will the money last?

- Salt brine water mixing problem.
- Newton's law of cooling. (You should have memorized the law of cooling.)
- Are there any other applications from class or from the homework?

2. SYSTEMS OF DIFFERENTIAL EQUATIONS

Before you were looking for solutions to a single differential equation, and also learning how to analyze the qualitative features of solutions in case you cannot find a solution explicitly.

- What is a system of differential equations?
- What is a solution to a system of differential equations?
- With our current tools, when can we solve systems of differential equations?
- What sort of information can we look for if we cannot solve the system of DE's explicitly?

2.1. Phase Diagrams.

- What are phase diagrams?
- How are they different from slope fields?
- What are v-nullclines, and how do you find them?

- What are h-nullclines, and how do you find them?
- What are equilibrium solutions, and how do you find them?
- When are solutions stable or unstable?
- Explain the following question in the context of systems of DE's: Can two species coexist?

2.2. Linear Algebra. You are learning about matrices so that you can develop tools to solve systems of differential equations. (You should recall how you solved systems of *LINEAR EQUATIONS* using matrices in the past.) It feels like a complete change of pace, but you will be headed back to DE's soon enough.

Define the following terms and explain how to carry out each of the matrix operations:

- matrix multiplication. (You cannot multiply any two matrices. Give an example of when you cannot do this.)
- scalar multiplication. Can you always do this? If not, give an example.
- matrix addition. Can you always do this? If not, give an example.
- taking the transpose of a matrix. Can you always do this? If not, give an example.

3. REVIEW

Math always builds on itself, so you will have to go back and remember from 3A and 3B.

- differentiation formulas
- integral formulas
- integral methods: u-substitution, integration by parts, partial fractions, and possible trig substitution
- implicit differentiation/chain rule
- basic trig formulas

4. STUDY TIPS

- Work practice problems
- Understand how all of the concepts fit together. Later problems from the homework force you to assimilate old and new ideas.
- Know how to work all of your homework problems and questions from previous midterms and quizzes. You wouldn't want to be caught making the same mistake twice.
- Memorize all formulas from class and homework.
- Keep in mind that you are likely to get problems that are not exactly like homework problems, because this tests the depth of your understanding of the subject.
- You should understand what you did in homework and why well enough that you immediately nail any "type problems" that you have seen before on homework/quizzes/midterms, and then move on to more critical thinking problems where you will have to be creative and flexible with your understanding.

Good luck everyone.

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