A pixel on your computer is like a little box of color made by combining different amounts of red ($r$), green ($g$), and blue ($b$). You can have red, green, and blue ranging from intensity 0 to intensity 255. (This is because computers are written in binary and with counting starting at zero, $256 = 2^8$, is a power of 2.) The color of a given pixel is described in a vector
\[ \vec{v} = \begin{pmatrix} r \\ g \\ b \end{pmatrix}. \]

Black is the absence of color, so $r = g = b = 0$. White would have full intensity of all colors, so $r = g = b = 255$.

**Problem 5.1.** Write your two favorite colors as a vector using the RGB color scheme, $\vec{v} = \begin{pmatrix} r_v \\ g_v \\ b_v \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} r_w \\ g_w \\ b_w \end{pmatrix}$.

**Problem 5.2.** What matrix would switch the roles of red and blue in a pixel? Let’s call this $P$. (No reason to get this wrong! You can check your work by multiplying $P$ times a color vector and see what happens.)

**Problem 5.3.** What has to be true about $r$, $g$ and $b$ for $\vec{u} = \begin{pmatrix} r_u \\ g_u \\ b_u \end{pmatrix}$ to be a grey pixel? Give an example.

**Problem 5.4.** Apply $P$ to $\vec{v}$, $\vec{w}$, and $\vec{u}$. Go ahead and write out the matrix-vector multiplication.
Notes on Matrix Multiplication

Is there a limit to how many pixel vectors you can multiply at a time?

**Problem 5.5.** Write your own matrix that gets rid of the blue component. Call it $Y$.

**Problem 5.6.** Without doing a calculation, describe what $YP$ does? Is it the same as $PY$? Check your guess with a calculation.

**Problem 5.7.** What does the matrix $G = \begin{pmatrix}
\frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\
\frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\
\frac{1}{2} & \frac{1}{3} & \frac{1}{4}
\end{pmatrix}$ do? If you’re not sure, see what is does to white, black, red, green and blue.

**Problem 5.8.** You are a tech person for the FBI analyzing seemingly endless recordings on your computer looking for a red car. (Some linear algebra thugs took it for a joy ride.) The image on your computer is stored in a large matrix of pixels like we discussed above. Find a matrix, $R$, that saturates red coloring to help you locate the red car. What matrix, $B$, would help you find his accomplice in the blue car?

**Problem 5.9. Bonus:** How would you use this to find the negative of an image?
Consider the following matrices:

\[
A = \begin{pmatrix}
1 & 1 & 2 \\
1 & 2 & 3 \\
\end{pmatrix}, \quad
B = \begin{pmatrix}
1 & 0 \\
1 & 2 \\
5 & 2 \\
\end{pmatrix}, \quad
C = \begin{pmatrix}
1 & -1 \\
1 & 2 \\
\end{pmatrix}, \quad
D = \begin{pmatrix}
2 & 0 & 0 \\
0 & 3 & 0 \\
0 & 0 & 1 \\
\end{pmatrix},
\]
\[
E = \begin{pmatrix}
1 & 0 & 2 \\
1 & 2 & -1 \\
0 & 0 & 4 \\
\end{pmatrix}, \quad
F = \begin{pmatrix}
a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\
a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m,1} & a_{m,2} & \cdots & a_{m,n} \\
\end{pmatrix} = (a_{i,j})_{1 \leq i \leq m, 1 \leq j \leq n}
\]

**Problem 5.10.** Evaluate the following or explain why they cannot be evaluated.

1. \(BC\)

2. \(CB\)

3. \(A^2B\)

4. \(D^2B\)

5. \(DE\)

6. \(ABC\)

7. \(A + 2B\)

8. \(D + 3E\)

9. \(A^T + 2B\)

10. \(B^T D\)
Problem 5.11. Check your understanding.

1. What must matrices $Q$ and $R$ satisfy for you to be able to add them?

2. What must matrices $Q$ and $R$ satisfy for you to be able to multiply them?

Problem 5.12. True or false: If $A$ and $B$ are matrices with $AB = 0$, then either $A = 0$ or $B = 0$. (Once upon a time, this was an exam question.)

Problem 5.13. The following page is an old study guide for linear algebra. At this point, I expect you could do all but 1.1 and 6-10.
Self-Assessment 1 – Math 3C, Fall 2011

Answer the following questions without looking in the book. If you do not feel comfortable doing this, read the corresponding sections in the book, and then solve the problems, again without looking in the book.

1. Given matrices $A$ and $B$ of dimensions $3 \times 6$ and $6 \times 4$, determine which one of the following expressions is well defined:
   
   (a) $AB$.
   (b) $A^T B$.
   (c) $AB^T$.
   (d) $A + B$.
   (e) $BA$.
   (f) $A^T B$.
   (g) $AA$.
   (h) $A^T A$.
   (i) $BB$.
   (j) $BB^T$.
   (k) $B^T B$.
   (l) $|A|$.

2. Given two $n \times n$ matrices $A$ and $B$, show the following, or give a counterexample:
   
   (b) $(A + B)^2 = A^2 + AB + BA + B^2$.
   (c) $(A + B)^2 = A^2 + 2AB + B^2$.
   (d) $(I + A)^2 = I + 2A + A^2$.

3. Is $(AB)^T = A^T B^T$? If not, find a counterexample.

4. Solve the following system of equations:
   
   \begin{align*}
   x + y + z + w &= 5 \quad (1) \\
   2x - y + 2z + 3w &= 6 \quad (2) \\
   4x - y - z &= 0 \quad (3)
   \end{align*}
5. Consider the following system of equations:

\[
\begin{align*}
  x - y - z &= 1 \\
  2x + 4y + z &= a \\
  x - 4y + bz &= 3
\end{align*}
\]

(a) Find all the values of \(a\) and \(b\) for which the system has a unique solution.

(b) Find all the values of \(a\) and \(b\) for which the system has infinitely many solutions.

(c) Find all the values of \(a\) and \(b\) for which the system does not have any solution.

6. What is the rank of a matrix?

7. Assume that an \(n \times n\) matrix \(A\) is invertible. Solve the system of equations \(Ax = 0\).

8. State the definition of determinant of an \(n \times n\) matrix.

9. Consider a square matrix \(A\), and assume that we have reduced the matrix to upper triangular form, \(U\), using Gauss elimination. Explain the relation between \(|A|\) and \(|U|\).

10. Given an \(n \times n\) matrix \(A\), explain why if \(|A^2| \neq 0\), then \(A\) is invertible.