Homework 4 due 10/22

Graded Problem: p 59 # 7

$\boxed{7}$ Prove that if $(v_1, \ldots, v_n)$ spans V and $T \in \mathcal{L}(V,W)$ is surjective (ie onto) then $(Tv_1, \ldots, Tv_n)$ spans W

Pf:

Suppose $(v_1, \ldots v_n)$ spans V and that $T \in \mathcal{L}(V,W)$ is surjective.

Let $w \in W$.

WTS: w can be written as a linear combination of $(Tv_1, \ldots, Tv_n)$.

ie that $w \in \text{Span}(Tv_1, \ldots Tv_n)$

Since T is surj., $\exists v \in V$ so that $Tv = w$.

We know $v = a_1 v_1 + \ldots + a_n v_n$ for n scalars $a_i \in \mathbb{F}$ because $\text{Span}(v_1, \ldots v_n) = V$.

$$
\begin{aligned}
w = Tv &= T(a_1 v_1 + \ldots + a_n v_n) \\
&= T(a_1 v_1) + \ldots + T(a_n v_n) \quad (T \text{ linear, additivity}) \\
&= a_1 Tv_1 + \ldots + a_n Tv_n \quad (T \text{ lin., homogen.})
\end{aligned}
$$

Then $w \in \text{Span}(Tv_1, \ldots, Tv_n)$

$\Rightarrow$ The Tv_i's span W.

$\blacksquare$