The problem needs:

Use Green's Thm to compute the area of one petal of the 24 leaved rose defined by $r = 20 \sin 12\theta$.

It may be useful to recall that the area of a region $D$ enclosed by a curve $C$ can be expressed as $A = \frac{1}{2} \int_C x\,dy - y\,dx$.

Notes:

1) You can calculate the area with $\iint_D 1\,dA$. If you start with $A = \frac{1}{2} \int_C x\,dy - y\,dx$, you can force an application of Green's Thm to move backwords to $\iint_D 1\,dA$. However, since the recalled formula for $A$ is a result of Green's Thm, this is like "undoing" the application of Green's Thm.

   See pg 2 for this calculation.

2) To apply Green's Thm in the "right" direction, calculate $x\,dy - y\,dx$ in polar coordinates. See page 3 for this calculation.

   - Now you can calculate area inside a curve in polar coordinates using this method.
   - This integral reduces to the same calculation as when you use $A = \iint_D 1\,dA$. 


HW 2 Problem 2

Question: Use Green's Thm to compute the area of a petal of the 24-leaf rose defined by $r = 20 \sin 12\theta$.

Recall: $A = \frac{1}{2} \int_C x \, dy - y \, dx$

Green's Thm $\int_C x \, dy - y \, dx = \frac{1}{2} \iint_D \left( \frac{\partial}{\partial x} x - \frac{\partial}{\partial y} (-y) \right) \, dA$

\[ r = 20 \sin 12\theta \]
\[ 0 \leq \theta < \frac{\pi}{12} \]

\[ \cos 2\theta = \cos^2 \theta - \sin^2 \theta = (1 - \sin^2 \theta) - \sin^2 \theta = 1 - 2 \sin^2 \theta \]
\[ \sin^2 \theta = \frac{1 - \cos 2\theta}{2} \]

\[ 12\theta \in [0, \pi/12] \]
\[ \theta \in [0, \pi/12] \]

Calculation:
\[ \iint_D 2 \, dA \]
\[ = \iint_D \sin^2 12\theta \, dA \]
\[ = \int_0^{\pi/12} \int_0^{20 \sin 12\theta} r \, dr \, d\theta \]
\[ = \frac{1}{2} \int_0^{\pi/12} \left[ r^2 \right]_{r=0}^{r=20 \sin 12\theta} \, d\theta \]
\[ = \frac{1}{2} \int_0^{\pi/12} 400 \sin^2 12\theta \, d\theta \]
\[ = 200 \int_0^{\pi/12} \frac{1}{2} \left( 1 - \cos 24\theta \right) \, d\theta \]
\[ = 100 \left[ \theta - \frac{\sin 24\theta}{24} \right]_0^{\pi/12} \]
\[ = 100 \left( \frac{\pi}{12} \right) \]
\[ = 25 \frac{\pi}{3} \]
\[ A = \frac{1}{2} \int_{c} \int \, dx \cdot dy - y \, dx \]

Calculate \( xy \, dx - y \, dx \) in polar coordinates.

\[ x = r \cos \Theta \implies dx = dr \cos \Theta + r(-\sin \Theta) \, d\Theta = \cos \Theta \, dr - r \sin \Theta \, d\Theta \]

\[ y = r \sin \Theta \implies dy = dr \sin \Theta + r \cos \Theta \, d\Theta = \sin \Theta \, dr + r \cos \Theta \, d\Theta \]

\[ x \, dy - y \, dx = (r \cos \Theta) [(\sin \Theta \, dr + r \cos \Theta \, d\Theta)] - (r \sin \Theta) [(\cos \Theta \, dr - r \sin \Theta \, d\Theta)] \]

\[ = \left[ r \cos \Theta \, \sin \Theta - r \sin \Theta \, \cos \Theta \right] \, dr - \left[ r^2 \, \cos^2 \Theta + r^2 \, \sin^2 \Theta \right] \, d\Theta \]

\[ = r^2 \, d\Theta \]

\[ A = \iint_{S} \, 1 \, dA = \frac{1}{2} \int_{c} \int \, dx \cdot dy - y \, dx \]

\[ = \frac{1}{2} \int_{c} r^2 \, d\Theta \\
= \int_{0}^{\pi/2} \frac{r^2}{2} \, d\Theta \]

\( \Theta \) still runs from 0 to \( \pi/2 \).

This is the same integral from the previous page once you plug in \( r = 20 \sin \Theta \).