A few highlights

• Suppose $f(x)$ is a periodic function with period $2\pi$ that can be represented by the following series and that series converges, then we have

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

where

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx \quad (1)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx \quad (2)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx \quad (3)$$

• Many functions can be represented by a Fourier series. For a list of requirements, see Theorem 2 in Section 11.1.

• The fundamental period of a function is the smallest $p$ so that $f(x + p) = f(x)$ for all $x$.

• Suppose $f(x)$ is a periodic function with period $p = 2L$ that can be represented by the following series and that series converges, then we have

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(\frac{n\pi}{L}x) + b_n \sin(\frac{n\pi}{L}x))$$

where

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x) dx \quad (4)$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos(\frac{n\pi}{L}x) dx \quad (5)$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin(\frac{n\pi}{L}x) dx \quad (6)$$
Practice Problems:

1. Find the fundamental period of the following functions:
   
   (a) $\cos(3x)$
   
   (b) $\sin\left(\frac{2\pi x}{k}\right)$
   
   (c) $\cos(2x) + \sin(3x)$

2. Show that a constant function $f(x)$ is periodic but has no fundamental period.

3. Graph the following periodic functions and give their Fourier representation. (The functions are defined to be periodic. Drawing the graph will help you understand what is meant by that.)

   (a) $f(x) = x^2 \ (-\pi < x < \pi)$
   
   (b) $f(x) = |\pi - x| \ (-\pi < x < \pi)$
   
   (c) $f(x) = \sin(\pi x) \ 0 < x < 1 \ p = 1$
   
   (d) $f(x) = 1 \ (1 < x < 3), f(x) = 0 \ (-1 < x < 1) \ p = 4$