Mathematics 3C Summer 3B 2009

Worksheet 9, September 8th, TA Grace Kennedy

NAME:___________________________________

PREP for 9/10: Finish worksheets, and STUDY STUDY STUDY!

MY WEBSITE: http://math.ucsb.edu/~kgracekennedy/SB093C.html

Write clearly and justify every step. Consider the following system of linear equations:

$$\begin{array}{cccccccc} x_1 & +2x_2 & +x_3 & +2x_4 & +3x_5 & = & 0, \\ x_1 & +2x_2 & +2x_3 & +3x_4 & +4x_5 & = & 0, \\ 2x_1 & +4x_2 & +2x_3 & +4x_4 & +6x_5 & = & 0. \end{array}$$

1. Write out the coefficient matrix this homogeneous system of linear equations.

2. Use the elementary row operations to put the matrix in row-reduced echelon form.

3. Find a basis for the row space $W \subseteq \mathbb{R}^5$ of the coefficient matrix, the subspace of $\mathbb{R}^5$ spanned by the rows of the original matrix. What is the dimension of W?

4. Find a basis for the space $W^\perp \subseteq \mathbb{R}^5$ consisting of the solutions to the original linear system. What is the dimension of $W^\perp$?

Consider the following system of two differential equations:

$$\begin{aligned} x'(t) &= x(t) + y(t) \\ y'(t) &= 4x(t) + y(t) \end{aligned}$$

(This is a linear system of differential equations.)
Let $\hat{v}'(t) = \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix}$ and $\hat{v}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$.

1. The coefficient matrix is the matrix A so that $\hat{v}'(t) = A\hat{v}(t)$. Write the coefficient matrix for this system. (You find this in exactly the same way as you would for a system of linear equations.)

2. You will see in 5A, that this matrix has two *eigenvalues*, $\lambda_1 = -1$ and $\lambda_2 = 3$, and that they are associated with two *eigenvectors* $v_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Are these vectors linearly independent? Why or why not?

3. On the back, check that

$$\begin{aligned} x(t) &= c_1 e^{-t} + c_2 e^{3t} \\ y(t) &= c_1(-2e^{-t}) + c_2(2e^{3t}) \end{aligned}$$

is a solution to the given system of differential equations.

In 5A, you will write the above solution as $\hat{v}(t) = c_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. In other words, your general solution consists of all of the vectors spanned by the linearly independent vectors $e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Your general solution is a vector space of dimension 2, and the two vectors above form a basis.