Worksheet 9

5.1.2 Find the eigenvalue of $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & 3 \end{bmatrix}$. Find a basis for each of the corresponding eigenspaces.

5.1.22b If $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent eigenvectors, do they necessarily correspond to distinct eigenvalues?

5.1.26 Show that if A^2 is the zero matrix, then the only eigenvalue of A is 0.

5.3.9 If possible, diagonalize the matrix

$$\begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}.$$

If not possible, explain why it cannot be done.

5.3.25 A is a 4×4 matrix with three eigenvalues. One eigenspace is one-dimensional, and one of the other eigenspaces is two dimensional. Is it possible that A is not diagonalizable? Justify your answer.

5.3.31-32 Construct (1) a nonzero 2×2 matrix that is invertable but not diagonalizable, and (2) a nondiagonal 2×2 matrix that is diagonalizable but not invertable.

You saw that the eigenvalues of a matrix A are the roots of the polynomial $\det(A - \lambda I)$. You have also seen that the rotation matrix $R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ has no real eigenvalues. What happens if you try to use the above formula to compute them? What do you make of this? Bonus: Must a 3×3 real matrix always have a real eigenvalue? What does this tell you about rotations in three dimensional space?