Worksheet 6
1, Lay: 1.9.13, 2.2.16, 3.1.9,3.2.5,3.2.32,3.2.35
1.9.13 Let $T: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$ be the linear transformation such that $T\left(\mathbf{e}_{1}\right)$ and $T\left(\mathbf{e}_{2}\right)$ are the vectors shown in the figure. Using the figure, sketch the vector $T(2,1)$.

2.2.16 Suppose $A$ and $B$ are $n \times n$ matrices, $B$ is invertable, and $A B$ is invertable. Show $A$ is invertable. [Hint: Let $C=A B$, and solve this equation for $A$ ].
3.1.9 Compute by cofactor expansion the determinant of
$\left|\begin{array}{cccc}6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8\end{array}\right|$.

### 3.2.5 Find by row reduction the determinant of

$$
\left|\begin{array}{ccc}
1 & 5 & -6 \\
-1 & -4 & 4 \\
-2 & -7 & 9
\end{array}\right|
$$

In other words, reduce this matrix to row echelon form and use the properties of how determinants transform under row operations.
3.2.32 Find a formula for $\operatorname{det}(r A)$ where $A$ is an $n \times n$ matrix.
3.2.35 Let $U$ be a square matrix such that $U^{T} U=I$. Show that $\operatorname{det} U= \pm 1$.

Suppose

$$
A=\left[\begin{array}{cc}
2 & 1 \\
-2 & 1
\end{array}\right]
$$

and

$$
B=\left[\begin{array}{ll}
6 & 1 \\
2 & 2
\end{array}\right]
$$

Find

$$
\operatorname{det} A^{2} B^{7} A^{-1}
$$

