Name:
Tardis:

Quiz 9

Consider the vector

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

Find the set of all vectors orthogonal to $\mathbf{v}$.

We wish to find the set of all vectors $\mathbf{x} \in \mathbb{R}^3$ such that $\mathbf{v} \cdot \mathbf{x} = 0$. In other words, the set of all vectors $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ such that

$$0 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 1x_1 + 2x_2 + 1x_3 = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},$$

i.e. the nullspace of $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$.

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$ is already in reduced row eschelon form and has two free variables, x_2, x_3. So the solution set to the homogeneous equation $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \mathbf{x} = 0$ (i.e. the nullspace) is given by

$$\begin{aligned} x_1 &= -2x_2 - x_3 \\ x_2 &= x_2 \\ x_3 &= x_3, \end{aligned}$$

or

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

We can also write the nullspace as

$$\text{span}\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \}.$$

And by the argument above, this nullspace is exactly the set of all vectors orthogonal to $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.