Name:
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Quiz 7

Is the set

$$\left\{ \begin{bmatrix} c - 6d \\ d \\ c \end{bmatrix} : c \text{ and } d \text{ are real numbers} \right\}$$

a vector space?

Yes. Here are two solutions:

This is the span of two vectors: $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -6 \\ 1 \\ 0 \end{bmatrix}$. The span of a set of vectors is always a vector space. (4.1 Theorem 1).

Or:

This set is a subset of $\mathbb{R}^3$. So we only need to check that it has a 0 vector and is closed under addition and scalar multiplication.

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is in our set (take $c = d = 0$).

It is closed under addition:

$$\begin{bmatrix} c_1 - 6d_1 \\ d_1 \\ c_1 \end{bmatrix} + \begin{bmatrix} c_2 - 6d_2 \\ d_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} (c_1 + c_2) - 6(d_1 + d_2) \\ d_1 + d_2 \\ c_1 + c_2 \end{bmatrix}.$$

And it is closed under scalar multiplication:

$$a \begin{bmatrix} c - 6d \\ d \\ c \end{bmatrix} = \begin{bmatrix} ac - 6ad \\ ad \\ ac \end{bmatrix}.$$