Name:
Tardis:

Quiz 5

Compute both the inverse and the determinant of the matrix

$$A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}.$$

The determinant of a 2×2 matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is simply $ad - bc$. This gives us

$$\det A = 1 * 2 - 0 * 3 = 2.$$

The inverse of such a 2×2 matrix is given by

$$\frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}. \tag{1}$$

This gives us

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

.

You should memorize (1). However, if you forgot it, you could also compute the inverse by combining A with I_2 to form a "super-augmented" matrix and row-reducing until the left half was the identity. The right half would then be the inverse of A. I.e.

$$\begin{bmatrix} 1 & 3 & | & 1 & 0 \\ 0 & 2 & | & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & | & 1 & 0 \\ 0 & 1 & | & 0 & \frac{1}{2} \end{bmatrix}$$
$$\sim \begin{bmatrix} 1 & 0 & | & 1 & -\frac{3}{2} \\ 0 & 1 & | & 0 & \frac{1}{2} \end{bmatrix}$$

and thus

$$A^{-1} = \begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & \frac{1}{2} \end{bmatrix},$$

exactly as we obtained with (1).