Name:
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Quiz 4

Are the three vectors

$$
\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix}
$$

linearly dependent? Why or why not?

We wish to see if the system corresponding to

$$
\begin{bmatrix} 1 & 3 & 3 & | & 0 \\ -1 & 0 & 1 & | & 0 \\ 1 & 6 & 7 & | & 0 \end{bmatrix}
$$

has nontrivial solutions. Let us add Row 1 to Row 2 and subtract Row 1 from Row 3. We obtain

$$
\begin{bmatrix} 1 & 3 & 3 & | & 0 \\ 0 & 3 & 4 & | & 0 \\ 0 & 3 & 4 & | & 0 \end{bmatrix}.
$$

Then subtracting Row 2 from Row 3,

$$
\begin{bmatrix} 1 & 3 & 3 & | & 0 \\ 0 & 3 & 4 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}.
$$

Observe that this system is consistent and has a free variable, $x_3$, and therefore this system has nontrivial solutions. Thus the set of three vectors is linearly dependent.

Alternatively, we might have observed that

$$
\begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix} = \frac{3}{4} \left( \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix} \right)
$$

and thus the three vectors are linearly dependent. However, such an insight requires inspiration and may not always be obvious.

1