Name:

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<div align="center">Quiz 3</div>

Let $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Suppose T is a linear transformation such that $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

and $T(\mathbf{v}) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$. What is $T(\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix})$?

Hint: $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = 2\mathbf{u} + \mathbf{v}$.

Lets make use of the hint to write

$$T(\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}) = T(2\mathbf{u} + \mathbf{v})$$

. Critically, T is a *Linear* transformation, so we can write

$$\begin{aligned} T(2\mathbf{u} + \mathbf{v}) &= T(2\mathbf{u}) + T(\mathbf{v}) \\ &= 2T(\mathbf{u}) + T(\mathbf{v}). \end{aligned}$$

But we know what $T(\mathbf{u})$ and $T(\mathbf{v})$ are! So this becomes

$$\begin{aligned} T(2\mathbf{u} + \mathbf{v}) &= T(2\mathbf{u}) + T(\mathbf{v}) \\ &= 2T(\mathbf{u}) + T(\mathbf{v}) \\ &= 2\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 7 \\ 4 \end{bmatrix}. \end{aligned}$$