Name:

Quiz 2

Let

$$A = \begin{bmatrix} 0 & 0 & 4 \\ 0 & -3 & -2 \\ -3 & 9 & -6 \end{bmatrix}$$

Do the columns of A span $\mathbb{R}^3$? ($\mathbb{R}^3$ is the set of all vectors with 3 components).
You do not need to do much computation, so justify your answer clearly and thoroughly.
Hint: Recall that the columns of a $n \times m$ matrix A span $\mathbb{R}^n$ if and only if for every vector $\mathbf{b}$ *in* $\mathbb{R}^n$*, the equation $A\mathbf{x} = \mathbf{b}$ is consistent.*

Let us make use of the hint (aka Theorem 4, Ch 1.4). Let us see that $A\mathbf{x} = \mathbf{b}$ is consistent, not matter what we choose for $\mathbf{b}$.

Choose $\mathbf{b}$ to be the vector

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

Then the system $A\mathbf{x} = \mathbf{b}$ corresponds to the matrix

$$\begin{bmatrix} 0 & 0 & 4 & | & b_1 \\ 0 & -3 & -2 & | & b_2 \\ -3 & 9 & -6 & | & b_3 \end{bmatrix}.$$

Switch R1 and R3:

$$\begin{bmatrix} -3 & 9 & -6 & | & b_3 \\ 0 & -3 & -2 & | & b_2 \\ 0 & 0 & 4 & | & b_1 \end{bmatrix}.$$

The matrix is now in row echelon form. Furthermore, it does not have any pivot in the last (augmented) column. Therefore it is consistent, no matter what we choose for b_1, b_2, b_3. (Alternatively, none of the rows have the form $\begin{bmatrix} 0 & 0 & 0 & | & b \end{bmatrix}$ where $b \neq 0$).