Name:

<div align="center">Quiz 1</div>

Consider the linear system associated with the augmented matrix

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 1 & 2 & 5 & 1 \\ -1 & 6 & -5 & 12 \end{array}\right]$$

Reduce this to row-echelon form. Is this system consistent?

The matrix we are given has a leading 0 in the first row, and it isn't part of a column of all 0s. We first want to switch rows so we have a pivot in the first entry of the first row.

Switch R1 and R2:

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 1 \\ 0 & 1 & 0 & 2 \\ -1 & 6 & -5 & 12 \end{array}\right].$$

Add R1 to R3 to eliminate the -1:

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 8 & 0 & 13 \end{array}\right].$$

Add -8R2 to R3 to eliminate the 8:

$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & 5 & 1 \\ 0 & \boxed{1} & 0 & 2 \\ 0 & 0 & 0 & \boxed{-3} \end{array}\right].$$

This matrix is now in row-echelon form (verify this). I have placed boxes around the pivots.

What is the interpretation of the third row as an equation? It tell us that

$$-3 = 0x_1 + 0x_2 + 0x_3 = 0.$$

But -3 doesn't equal 0, no matter what we choose for x_1, x_2, x_3. So the system is inconsistent – it can have no solutions. ∎