Activity 10: Practice Final

These problems focus on the **new material only.** You should also go over the previous midterms and practice midterms to prepare.

Question. *Is the following set of vectors orthogonal?*

$$\begin{bmatrix} 2 \\ -7 \\ -1 \end{bmatrix}, \begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$$

Question. *Find a least squares solution of $Ax = b$ by constructing the normal equations for $\hat{x}$ and solving for $\hat{x}$ with $A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$.*

Question. *Diagonalize the following matrix M by finding D, P and P^{-1} so that $M = PDP^{-1}$:*

$$M = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$

Question. *Use your answer to the previous problem to find a formula for M^n where n is an arbitrary integer.*

Question. *If* $T : \mathbb{R}^n \to \mathbb{R}^m$ *is a linear transformation and* $\{v_1, v_2, v_3\}$ *is linearly dependent in* $\mathbb{R}^n$*. Why is the set* $\{T(v_1), T(v_2), T(v_3)\}$ *linearly dependent in* $\mathbb{R}^m$*?*

Question. *Find the inverse of* $\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$ *and use it to solve the system:*

$$\begin{aligned} 8x_1 + 6x_2 &= 2 \\ 5x_1 + 4x_2 &= -1 \end{aligned}$$

(You must use the inverse. Do not RREF the system of equations.)

Question. *Let W be the first and third quadrants in the plane, i.e. $W = \{(x, y)|xy \geq 0\}$.*

- *Is W closed under scalar multiplication? Why or why not?*
- *Find two vectors, u and v in W, so that $u + v$ is not in W.*
- *Is W a subspace of $\mathbb{R}^2$?*

Question. *Let W be the set of all vectors of the form $\begin{bmatrix} 2b + 3c \\ -b \\ 2c \end{bmatrix}$ where b and c are arbitrary. Find vectors u and v so that $W = Span\{u, v\}$. Why does this show that W is a subspace of $\mathbb{R}^3$?*

Question. *Find Col A and Nul A:*

$$A = \begin{bmatrix} 1 & -4 & 0 & 2 & 0 \\ 0 & 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

Question. *Find A such that the given set is Col A:*

$$\left\{ \begin{bmatrix} 2s+t \\ r-s+2t \\ 3r+s \\ 2r-s-t \end{bmatrix} : r, s, t \ \textit{real} \right\}$$

Question. *Is the following set a basis for* $\mathbb{R}^3$*? Fully justify your answer.*

$$\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

Question. *Find the vector* x *determined by the given coordinate vector* $[x]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$ *and the basis* $\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ -5 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix} \right\}$

Question. *Find the coordinate vector* $[x]_{\mathcal{B}}$ *of* $x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ *relative to the basis* $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \end{bmatrix} \right\}$

Question. $T : \mathbb{R}^2 \to \mathbb{R}^2$ *is a linear transformation that first reflects across the line* $x = y$ *and then rotates by* 90 *degrees clockwise. Find the standard matrix for* T*, and it's eigenvalues and eigenvectors.*

Question. *Find the eigenvalues and eigenvectors for the matrix* $\begin{bmatrix} 3 & 1 & 1 \\ 0 & 5 & 0 \\ -2 & 0 & 0 \end{bmatrix}$

Question. *Define linear independence, span, basis, eigenvalue, eigenvector, orthogonal and orthonormal.*

Question.

- *Find a unit vector in the direction of* $u = \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$.
- *Find a unit vector orthogonal to* u.