Handout 1

September 30, 2015

Problem 1 Find the general solution to the equation:

$$\frac{dy}{dx} = \frac{y}{x} + 2x + 1$$

Problem 2 Find the general solution to the equation:

$$\frac{dr}{d\theta} + r\tan(\theta) = \sec(\theta)$$

Problem 3 The equation

$$\frac{dy}{dx} + 2y = xy^{-2}$$

is an example of a Bernoulli Equation.

- Show that the substitution $v = y^3$ reduces the equation to

$$\frac{dv}{dx} + 6v = 3x$$

- Solve the new equation for v. Then make the substituion $v = y^3$ to obtain the solution to the original Bernoulli equation.

Problem 4 Consider the Bernoulli Equation:

$$\frac{dy}{dx} + p(x)y = q(x)y^n$$

For $n = 0$ or $n = 1$ the equation is linear and we know how to solve it. Solve the Bernoulli Equation. (Hint: try the substitution $v = y^{1-n}$)

Method for Solving Linear Differential Equations

1. Write the equation in standard form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

2. Calculate the integrating factor $\mu(x)$:

$$\mu(x) = \exp[\int P(x)dx]$$

3. Multiply the equation in standard form by $\mu(x)$:

$$\mu(x)\frac{dy}{dx} + P(x)\mu(x)y = \mu(x)Q(x)$$

   which then simplifies to

$$\frac{d}{dx}[\mu(x)y] = \mu(x)Q(x)$$

4. Integrate and divide by $\mu(x)$.