

MATH 34A REVIEW FOR MIDTERM 1, SPRING 2011

ANSWERS

1. ALGEBRA

- (1) Simplify $(x + \frac{1}{x})(x^2 + 1)^{-1} + \frac{1}{x}$.

The expression equals $2/x$.

- (2) Solve for x : $\frac{2x+d}{x+1} = a$.

$$x = \frac{a-d}{2-a}.$$

- (3) Solve the following system of equations:

$$2x + 3y = a$$

$$x + y = b$$

The answer is $x = 3b - 1, y = a - 2b$.

- (4) There are two positive numbers. Four times the small number plus 3 times the big number is 46. Two times the small number plus the big number is 19. What is 10 times the big number minus 6 times the small number?

The two numbers are $11/2$ and 8. So the answer is 47.

- (5) I have three numbers. The biggest one is twice the middle one, and the biggest one plus the middle one is four times the smallest one. The smallest one plus the middle one is two less than the biggest one. What are the numbers?

The numbers are 6, 8, and 16.

2. GEOMETRY

- (1) A circular can has height h and the base is a circle with radius R . If the volume is 4π express R in terms of h .

$$R = 2h^{-\frac{1}{2}}.$$

- (2) The perimeter of a rectangle equals $3/2$ times its area. Express the length of the rectangle in terms of the width.

$$l = \frac{2w}{(3/2)w-2}.$$

- (3) A window has the shape of a semi-circle placed on top of a square. The glass costs \$7 per square meter. Write the total cost of the window in terms of the radius of the circle.

$$\text{Total cost} = 7\left(\frac{\pi}{2} + 4\right)r^2 \text{ dollars.}$$

- (4) What is the distance between the points $(-3,8)$ and $(5,2)$? Draw a picture.

Draw the distance as the hypotenuse of a right triangle. Then the Pythagorean Theorem tells us

- (5) What is the length of the hypotenuse of a right angled triangle when the other two sides have lengths 5 and 12?

The length is 13.

- (6) The vertical mast of a yacht is 40 feet high. A rope runs in a straight line from the top to a pulley 30 feet horizontally from the base of the mast. How many feet long is the rope?

50 feet.

3. UNITS

- (1) If the units of A is dollars/meters and the units of B is meters. What are the units of A/B and AB ?

A/B has units dollars per square meter AB has units dollars.

- (2) A swimming pool is 3 meters deep, 5 meters wide, and 80 meters long. Water is pumped into the pool at a rate of 100 liters per minute. How many hours will it take to fill the pool? (Remember that there are 1000 liters in a cubic meter).

200 hours.

- (3) If a sphere has radius 7 inches, what is the volume of the sphere in pints? The volume of a sphere is $\frac{4}{3}\pi r^3$. Also use that 1 foot = 12 inches, $1 \text{ ft}^3 = 7.5$ gallons, and 1 gallon = 8 pints. You do not need to multiply out.

The volume of the sphere is $\frac{36\pi}{12^3}(7.5)(8)$ pints.

- (4) If the radius of the sphere is doubled, the volume is how many times as big?

$2^3 = 8$ times as big.

4. PERCENTAGES

- (1) A manager starts with a salary of 50,000 dollars. After one year he received a 25% pay rise. After another year his pay is cut by 10%. What is his salary after this?

\$56,250

- (2) If 1000 liters of water with an unknown chlorine content are combined with 500 liters of water with 45 ppm (parts per million) of chlorine the result is 75 ppm chlorine. What is the concentration of chlorine in the unknown water?

90 ppm.

- (3) There are 3 beakers each of which contains saline solution. Beaker A initially contains 3 liters of 10% salt solution. Beaker B initially contains 2 liters of 20% salt solution. Beaker C initially contains 4 liters of 0% salt solution. Two liters are transferred from A to B and the result is thoroughly mixed. Then one liter is transferred from B to C and the result mixed. Finally two liters are transferred from C back to A. What is the percentage concentration of salt in A after all this?

$15/3$

- (4) I have milk that contains 1 percent fat and milk that contains 4 percent fat. A customer wants a double latte made with $\frac{1}{2}$ of a pint of 2 percent milk. How much of each type of milk should I use?

You should use $\frac{1}{3}$ pints of 1 percent milk and $\frac{1}{6}$ pints of 4 percent milk.

- (5) If I combine x liters of blue paint with y liters of red paint what percentage of blue paint is in the combination?

$\frac{x}{x+y}(100\%)$

5. FUNCTIONS

- (1) What is the inverse of the function $f(x) = \frac{a}{x}$?

$f^{-1}(x) = \frac{a}{x}$.

- (2) What is the inverse of the function $f(x) = 2x - 1$?

$f^{-1}(x) = \frac{x+1}{2}$.

- (3) What is the inverse of the function $f(x) = \frac{2x+1}{3x-5}$?

$f^{-1}(x) = \frac{5x+1}{3x-2}$.

- (4) What is the inverse of the function $f(x) = 8x^3$?

$f^{-1}(x) = \frac{x^{1/3}}{2}$.

- (5) The function $f(x)$ converts degrees Celsius to degrees Fahrenheit. What does the inverse function do? Recall that $f(x)$ is a line. Further recall that water freezes at 0 degrees Celsius or 32 degrees Fahrenheit and boils at 100 degrees Celsius or 212 degrees Fahrenheit. Derive a formula for $f(x)$. What is $f(30)$?

$$f(x) = \frac{9}{5}x + 32.$$

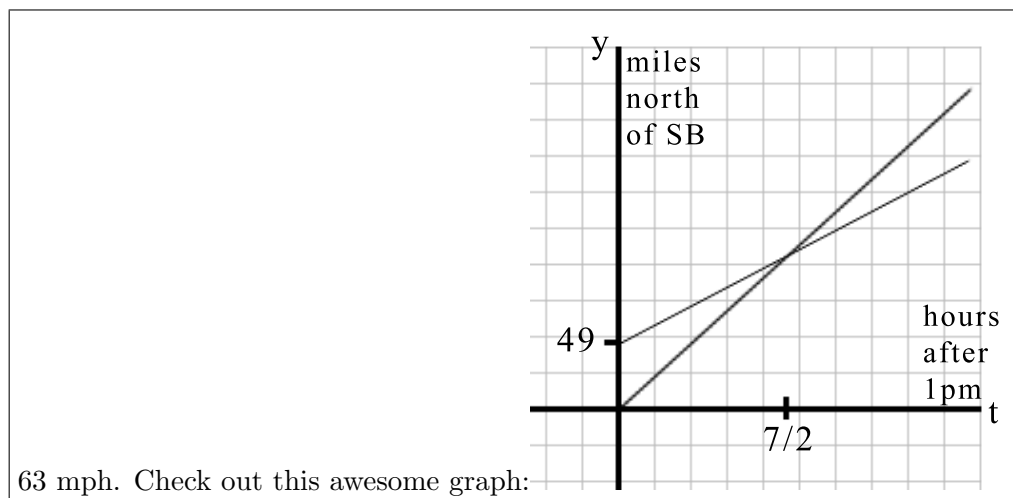
$f(30) = 86$ degrees Fahrenheit.

6. CAR PROBLEMS

- (1) SB and LAX are 120 miles apart. The Santa Barbara airbus leaves LAX at 3pm and drives to UCSB at an average speed of 30 mph. You leave UCSB at 4pm driving at 90 mph towards LAX. What time do you pass the airbus?

4:45pm

- (2) Car A leaves SB at noon driving north at 49 mph. Car B leaves at 1 pm traveling the same route at a different constant speed. How fast should Car B go in order to catch up with Car A by 4:30pm?



- (3) A highway patrolman traveling at the speed limit is passed by a car going 15 mph faster than the speed limit. After one minute, the patrolman speeds up to 90 mph. How long after speeding up until the patrolman catches up with the speeding car. The speed limit is 65 mph.

8 minutes.

- (4) Suppose Car A's distance in miles north of SB t hours after noon is given by the function $A(t) = 300 - 90t$ and Car B's distance in miles north of SB t hours after noon is given by the function $B(t) = 60t$. Describe the scenario. Calculate when and where the cars meet.

At noon Car A starts 300 miles north of SB and travels 90 mph south. At the same time, Car B starts in SB and travels 60 mph north.

They meet at 2pm 120 miles north of SB.

7. ERROR AND LIMITS

- (1) If you think a carton of milk is $1/5$ full but it is really $1/4$ full, what is the percentage error?

The percentage error is 20 percent.

- (2) You have \$55 in your pocket, but you think you have \$60. What is the percentage error?

The percentage error is $100/11$ percent.

- (3) Calculate the following limit: $\lim_{n \rightarrow \infty} \frac{5}{n^3}$.

0

- (4) Calculate the following limit: $\lim_{x \rightarrow 3} \frac{x^3 - 2x - 3}{x - 3}$.

$$\lim_{x \rightarrow 3} \frac{x^3 - 2x - 3}{x - 3} = \lim_{x \rightarrow 3} x^2 + x = 12.$$

(5) Calculate the following limit: $\lim_{x \rightarrow \infty} (x^{-2} + 2)$.

$\boxed{2}$.

8. CHANGE OF A FUNCTION AND SUMMATION

(1) If x is increased from 6 to 7 how much does $\frac{1+x}{2+x}$ change by? Does the function increase or decrease when x goes from 6 to 7?

The function changes by $\boxed{1/72}$.

The function increases.

(2) (a) Find

$\frac{\text{the change in } (x-2)(x+2) \text{ as } x \text{ increases from } 1 \text{ to } 1+h}{h}$

where h is a positive real number. Simplify your answer.

$\boxed{h+2}$

(b) Find

$\lim_{h \rightarrow 0} \frac{\text{the change in } (x-2)(x+2) \text{ as } x \text{ increases from } 1 \text{ to } 1+h}{h}$

$\boxed{2}$

(c) Calculate $\sum_{i=1}^n 3$

$\boxed{3n}$

(d) Calculate $\sum_{n=1}^4 (n-1)$

$\boxed{6}$