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A surgery formula for the smooth Yamabe invariant

The smooth Yamabe invariant of a compact manifold M is defined as

$$\sigma(M) := \sup \inf \int_M \mathrm{scal}^g \; dv^g$$

where the supremum runs over all conformal classes $[g_0]$ on M and the infimum runs over all metrics g of volume 1 in $[g_0]$. The invariant $\sigma(M)$ is positive iff M carries a metric of positive scalar curvature.

We prove: If N is obtained by surgery of codimension ≥ 3 from M, then

$$\sigma(N) \geq \min\{\sigma(M), \Lambda_n\},$$

where $\Lambda_n > 0$ only depends on $n = \dim M$. This formula unifies and generalizes previous formulas by Gromov-Lawson, Schoen-Yau, Kobayashi, Petean-Yun and allows many conclusions by using bordism theory. In particular, bordisms classes represented by manifolds with $\sigma(M) > \epsilon$ form a subgroup in the corresponding bordism group.

The subgroup structure might be a first step to build a topological characterization of manifolds with $\sigma(M) > \epsilon$, extending the topological knowledge about the class of manifolds admitting positive scalar curvature metrics.