Review Sheet: Chapter Three

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LINEAR ALGEBRA

- Matrix Terminology & Corresponding Properties: Dimension (Order), Element (Entry), Row Vector, Column Vector, Square Matrix, Diagonal Element, Augmented Matrix, Reduced Row Echelon Form (RREF), Inverse Matrix
- Special Matrix & Corresponding Properties: Zero Matrix, Diagonal Matrix, Identity Matrix
- Operation for Single Matrix & Corresponding Properties: Scalar Multiplication, Determinant, Elementary Row (Column) Operations,
- Matrix Arithmetic (Between Matrices) & Corresponding Properties: Addition, Subtraction, Multiplication, Division
- Be Able to Solve Systems of Linear (Algebraic) Equations by Gaussian Elimination for Different Cases: Unique Solution, Infinitely Many Solutions, No Solution; Understand Correspondence Between Pivot, Rank, Invertibility, Singularity & Number of Solutions; Be Able to Solve Linear Systems by Nonhomogeneous Principle & Cramer's Rule
- Vector Space (Ten Conditions), Subspace (Two Conditions) & Their Characterization: Basis & Dimension

PRACTICE PROBLEMS

1

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & -2 \\ 0 & 1 & 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 & 1 & -1 \\ -1 & 1 & 3 \\ 1 & -1 & 1 \end{pmatrix},$$

and $k = 1/4$, calculate $\mathbf{B} = k\mathbf{C}$, $\mathbf{A} + \mathbf{C}$, $\mathbf{AB}$, $\mathbf{BA}$, $|\mathbf{A}|$, $|\mathbf{AB}|$, $|\mathbf{BA}|$, $\mathbf{A}^{-1}$.

2 Solve the following linear systems by Gaussian elimination, nonhomogeneous principle, and Cramer's rule (if possible)

$$\begin{aligned} x + z &= 2 \\ 2x - 3y + 5z &= 4 \\ 3x + 2y - z &= 4 \end{aligned}$$

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$$\begin{aligned} x + 2y + z &= 2 \\ 2x - 4y - 3z &= 0 \\ -x + 6y - 4z &= 2 \\ x - y &= 4 \end{aligned}$$

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$$\begin{aligned} x_1 + 2x_3 - 4x_4 &= 1 \\ x_2 + x_3 - 3x_4 &= 2 \end{aligned}$$

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3 Show that the set $C^1(\mathbb{R}) = \{\text{all continuously differentiable functions over } \mathbb{R}\}$ is a vector space. Show that the following sets:

$$V = \left\{y(t) | y(t) \in C^1(\mathbb{R}), \text{ and } y(1) = 0\right\},$$

and

$$W = \left\{y(t) | y(t) \in C^1(\mathbb{R}), \text{ and } y' + ty = 0\right\}$$

are subspaces of $C^1(\mathbb{R})$.

4 Show that $\mathbf{S} = \{[1, 0, 0], [1, 1, 0], [1, 1, 1]\}$ forms a basis of $\mathbb{R}^3$.