Review Sheet: Chapter One & Two

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DIFFERENTIAL EQUATIONS

- Modeling (Single Differential Equation): Malthus Model for Population Growth & Saving (Growth Equation), Newton's Law of Cooling & Radioactive Decay (Decay Equation), Saving & Mixing (Nonhomogeneous Equation), Logistic Equation & Threshold Equation (Nonlinear Autonomous Differential Equation)

- Modeling (System of Differential Equations): Predator-Prey Model & Competition Model (Autonomous System)

- Characterize Differential Equations : Single vs. System, Linear vs. Nonlinear, Order, Homogeneous vs. Nonhomogeneous, Autonomous vs. Non-autonomous, Separable vs. Nonseparable

- Qualitatively Analysis (Single Differential Equation): Direction Fields, Stability of Equilibria, Bifurcation Diagram

- Qualitatively Analysis (System of Differential Equations): Phase Portraits, Stability of Equilibria (Not Covered in 3C)

- Quantitative Analysis (Single Differential Equation): Explicit Solution, e.g., Separation of Variables & Integrating Factor Method (Euler-Lagrange Two-Stage Method); Numerical Solution, e.g., Euler Method & Runge-Kutta Method. Techniques From 3A & 3B Are Required, e.g., Differentiation & Integration, Integration by Parts, Substitution, Partial Fractions Decomposition....

- Quantitative Analysis (System of Differential Equations): Explicit Solution (Not Covered in 3C) & Numerical Solution (Not Covered in 3C)

- Existence & Uniqueness (Single Differential Equation): Picard's Existence & Uniqueness Theory
- Existence & Uniqueness (System of Differential Equations) (Not Covered in 3C)

PRACTICE PROBLEMS

1 Solve the logistic population model

$$\frac{dy}{dt} = ky(1-y),$$

where $k > 0$ is the growth rate constant. Draw the corresponding direction fields when $k = 1$ and analyze the stability of equilibria. What happens in the long run ($t \to \infty$)?

2 Solve the following differential equations

$$\begin{aligned} y' = -\frac{1+y^2}{1+t^2}, & \qquad y(0) = -1 \\ 3y^2y' - 2y^3 - t - 1 = 0, & \qquad y(0) = 2 \\ y' &= t^2 \exp(y+2t) \\ y' - y &= \exp(3t) \\ y' &= 1 - y^2 \\ y' &= y(a - b\ln y) \end{aligned}$$

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