

Differential Equations and Linear Algebra

Math 3C, Spring 2011

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Answer the following 3 questions. **Calculators are not allowed. The use of books or notes of any kind is not allowed. Show all your work for full credit.**

Name: _____

Perm Number: _____

Problem 1: _____ out of 6.

Problem 2: _____ out of 10.

Problem 3: _____ out of 9.

Total: _____ out of 25.

THESE SHEETS ARE TO BE HANDED IN WITH YOUR EXAM. THE GRADED EXAM CAN BE PICKED UP FROM MY OFFICE (DURING OFFICE HOURS).

1. (6 points) Answer the following questions indicating whether the statements are true or false (enclose the answer with a circle):
1. Every nonzero square matrix has an inverse (with respect to the product of matrices): True False
 2. Given an $n \times n$ matrix, if $\mathbf{Ax} = 0$ only has the zero solution, then $\mathbf{Ax} = \mathbf{b}$ has a unique solution for any $\mathbf{b} \in \mathbb{R}^n$: True False
 3. The product of matrices is associative: True False
 4. The product of matrices is commutative: True False
 5. Given any two matrices, their product is well defined: True False
 6. Given any two $n \times n$ matrices, \mathbf{A} and \mathbf{B} , $(\mathbf{AB})^T = \mathbf{A}^T\mathbf{B}^T$: True False

2. (10 points) Consider the following system of equations:

$$\begin{aligned}x - y - z &= 1 \\2x + 4y + z &= a \\x - 4y + bz &= 3\end{aligned}$$

1. Find all the values of a and b for which the system has a unique solution.
2. Find all the values of a and b for which the system has infinitely many solutions.
3. Find all the values of a and b for which the system does not have any solution.

3. (9 points) Consider the following set:

$$V = \{y \in C^1(\mathbb{R}) \mid y(0) = 0\}.$$

Show that V is a vector space over \mathbb{R} . (You can assume as known the fact that $C^1(\mathbb{R})$ is a vector space)