§2.4 Linear Models : Mixing and Cooling

Mixing Model: (equation of continuity)

If $x(t)$ is the amount of dissolved substance, then

$$\frac{dx}{dt} = \text{RATE IN} - \text{RATE OUT}$$

where RATE IN = (CONCENTRATION IN) (FLOW RATE IN)

RATE OUT = (CONCENTRATION OUT) (FLOW RATE OUT)

[lb/min] [lb/gal] [gal/min]

Ex : $\begin{cases} \frac{dx}{dt} = 4 - 0.1x \\ x(0) = 0 \end{cases}$

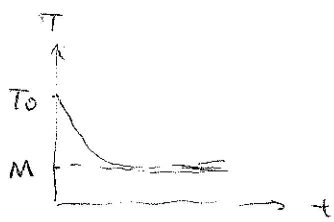
$x(t) = 40(1 - e^{-0.1t})$

Newton's Law of Cooling

The rate of change in the temperature T of an object placed in surroundings of uniform temperature M is porporational to the diffenence between the temperature of the object and the temperature of the surroundings.

Mathematically, $\frac{dT}{dt} = k(M-T)$

where $k>0$ is a constant of proportionality.



$\begin{cases} \frac{dT}{dt} = k(M-T) \\ T(0) = T_0 \end{cases}$ $T(t) = T_0 e^{-kt} + M(1-e^{-kt})$

Change of variable $y = M-T$, we have

$\begin{cases} \frac{dy}{dt} = -ky \\ y(0) = M - T_0 \end{cases}$ decay equation