§ 1.1 Dynamical Systems: Modeling

Differential Equations. the major interface of Mathematics with the real world

$\begin{cases} \text{continuous - time} \longrightarrow \text{differential equation} \\ \text{discrete - time (sampled - data)} \longrightarrow \text{iterative equation} \end{cases}$

Dynamical systems: systems that change over time

Def: A differential equation (DE) is a relation between a function and its derivatives, and independent variables

- An ordinary differential equation (ODE) contains only ordinary derivatives
- A partial differential equation (PDE) contains partial derivatives

The ORDER of a DE refers to the highest-order derivative that appears in the equation

Ex: ① $\frac{dy}{dx} = f(x)$

② $x(t)$: $\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = f(t)$ (forced damping spring)

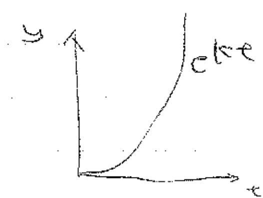
③ $y(t)$: $y' = y(1-y^2)$ nonlinear spring

④ temperature of a metalic rod $u(x,t)$

Fourier's law: $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$

$\frac{\partial}{\partial x}$: partial derivative w.r.t. x

⑤ Malthus Model for Population Growth



$y(t)$: population $\frac{dy}{dt} = ky$ $y = e^{kt}$

where k is called the growth or rate constant. $k > 0$

⑥ Hooke's Law : F_{res} : restoring force

$F_{res} = -kx$, $k > 0$ elastic constant

x deformation of the spring

Neglecting friction, Newton's law of motion gives us

$$F_{res} = m\frac{dv}{dt} = m\frac{d^2x}{dt^2} = -kx$$

Hooke's law as a System

$$\begin{cases} \dfrac{dx}{dt} = v \\[2mm] \dfrac{dv}{dt} = -\dfrac{k}{m}x \end{cases} \Rightarrow \frac{d}{dt}\begin{pmatrix} x \\ v \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{pmatrix}\begin{pmatrix} x \\ v \end{pmatrix}$$

In general, the ODE we will study is

$$y' = f(t, y)$$