

## Homework 2 – Math 104A, Fall 2011

Due on Tuesday, October 11, 2011

**Section 2.1:** 6, 12, 14, 16, and 20.

**Section 2.2:** 1, 2, 3, 4, 11.a, 11.b, 11.c, and 23.

**Additional problem:** Consider the iteration in Newton's method:

$$p^{n+1} = g(p^n)$$

where

$$g(x) = x - \frac{f(x)}{f'(x)}$$

Prove that if  $f(x^*) = 0$ , and  $f'(x^*) \neq 0$ , then  $g'(x^*) = 0$ . Deduce from this that the convergence of Newton's method is quadratic, i.e., if we denote by  $e_n$  the error of the  $n$ -th approximation ( $e_n = p^n - x^*$ ), then

$$e_{n+1} \leq K e_n^2$$

for some  $K > 0$ , as long as the initial iterate  $p^0$  is sufficiently close to  $x^*$ . You may assume that  $f$  has infinitely many continuous derivatives.