Designing DE's

Let's make up some DE's. Then we may get some insight into how to solve them.

1. Suppose f is a function such that $\ln(f(x)) = 2x$. Now use the chain rule to find a formula for the slope of a tangent line to any point (r, s) on the graph of f. You have created a differential equation. Find all its solutions.

2. Suppose f is a function such that $(f(x) + 1)^2 = \sin(x)$. Write a differential equation for which f must be a solution. Find all the solutions to the DE.

3. Find solutions to the following DE's

   (a) $2f'(x)f(x) = x^2$

   (b) $-f'(x) = xf(x)^2$

4. The circle of radius r centered at (a, b) is described in the plane by the equation $(x - a)^2 + (y - b)^2 = r^2$.

   (a) This equation does **not** define a function. Why?

   (b) But it can be used to define several functions *implicitly*. What does this mean?

   (c) Use the chain rule to find the slope of a tangent line at a point (r, s) of the graph of one of these implicit functions.

5. The equation $y^2 = x^3 - x$ has as graph a famous elliptic curve.

   (a) Give a formula for the slope of a tangent line to any point (r, s) of this curve.

   (b) Where are the tangents vertical?

   (c) Suppose a function $f(x)$ is implicitly defined by $y^2 = x^3 - x$. What differential equation does $f(x)$ satisfy?

6. Use the ideas you developed so far to find some solutions to the following differential equations. Use the chain rule in your explanations.

(a) $f'(x) = \frac{x+1}{f(x)-1}$

(b) $f'(x) = x(1 + f(x))^2$

(c) $f'(x) = e^{x+f(x)}$

7. Which DE's from assignment one can be solved this way? Solve them. DE's that can be solved by doing "u substitution" (i.e. the reverse of the chain rule) are called separable – why?