Assignment One

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My differential equation is $p'(x) = \sin(kp(x))$. I chose the constant $k = 1$. Another way of writing this D.E. is $\frac{dy}{dx} = \sin(y)$, and that is how I entered it into the slope field applet.

Long term behavior of my D.E. can be described by answering the questions at the top of page 23:

1. There appear to be many constant solutions. The first one I noticed was $p(x) = 0$. I checked to see that this was a solution by seeing if the left hand side and right hand side of the DE matched. To find the others, we assume that $p'(x) = C$ is a solution to the D.E. Then $p'(x) = 0 = \sin(C)$. This means C is a multiple of π. So the constant solutions for my DE are $p(x) = n\pi$ where n is any integer.

2. The DE is defined everywhere since the sine function is defined everywhere.

3. The only straight line solutions are the constant ones. We can see this because a straight line solution would be in the form $p(x) = mx + b$. So if there were such a solution then $p'(x) = m = \sin(mx + b)$ for all x and that is impossible unless $m = 0$.

4. The solution is concave whenever $p''(x) < 0$ and convex whenever $p''(x) > 0$ so first we must calculate $p''(x)$ by taking derivatives of both sides of the DE.
$$p''(x) = \cos(p(x))p'(x) = \cos(p(x))\sin(p(x))$$
The concavity switches when $p''(x) = 0$. In other words, the concavity switches whenever $\cos(p(x)) = 0$ or $\sin(p(x)) = 0$. Thus, the concavity switches back and forth as we cross the horizontal lines $p(x) = n\frac{\pi}{2}$ where n is an integer.

5. As x approaches ∞, $p(x)$ approaches a constant solution $y = 2n\pi$ for some integer n. These constant solutions(the even multiples of π) are stable for this reason.

6. As x approaches $-\infty$, $p(x)$ approaches a constant solution $y = (2n + 1)\pi$ for some integer n. These constant solutions (the odd multiples of π) are unstable for this reason.

7. The solutions are periodic in the sense that they repeat over and over across the plane. There are also asymptotes at the constant solutions.

Euler's method produced the picture attached.

To check the first two iterations by hand, we use the fact that

$$p(x+h) \approx p(x) + hp'(x)$$

Since $p'(x) = \sin(p(x))$ we rewrite the above as

$$p(x+h) \approx p(x) + hsin(p(x))h$$

Plugging in initial condition $p(0) = 0$ and $h = .1$, we get

$$p(0+.1) \approx p(0) + .1sin(p(0)) = 0$$

Similarly, we get $p(.2) = 0$. This corresponds to the constant solution $y = 0$.

For the initial condition $p(0) = \pi/2$, we get

$$p(.1) \approx \pi/2 + .1\sin(\pi/2) = \pi/2 + .1 \approx 1.67$$

$$p(.2) \approx p(.1) + .1\sin(p(.1)) \approx .1 + \pi/2 + .1\sin(.1 + \pi/2) \approx 1.77$$

We can double check this with the applet and the solution curve shown passing through $(0, \pi/2)$.

As I changed the constant, the constant solutions changed. For instance, picking k large packed the constant solutions closer together and picking k small spread the constant solutions out. Changing the sign of k switched the stable and unstable solutions.

As I decreased the step size for Euler's Method, the picture became better since, there were more points to connect. As I increased the step size the approximation to the solution looked more jagged but showed the general behavior of the solution. An example of a differential equation that "break" the applet is $y' = y^2$. It produces odd looking solutions that seems to go off to infinity.

The reason $y' = y^2$ isn't well-approximated by Euler's Method is that the slopes become very steep very quickly as y gets larger, causing the method to grossly overshoot higher points. The RKF method worked best since it simply did not display solutions for large y-values.