# Parallelograms and Determinants of 2x2 matrices

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We can think of a parallelogram as being defined by two vectors.

1. For example: Draw and find the area of the parallelogram spanned by the vectors $(5, 7)$ and $(2, 3)$

2. More examples and searching for patterns: Draw and find the areas of the parallelograms spanned by:
   $(0, 2)$ and $(1, 0)$
   $(1, 2)$ and $(1, 0)$
   $(1, 2)$ and $(2, 2)$
   $(3, 6)$ and $(6, 6)$
   Write down any patterns you notice.

3. Discuss with everyone your strategies for finding areas of parallelograms.

4. General case: Find the area of a parallelogram spanned by $(a, b)$ and $(c, d)$.

5. What does this have to do with the determinant of a matrix?

6. What "operations" can we do to a parallelogram that preserve its area (to a matrix to preserve its determinant) ?

7. What "operations" change the area and how? What happens if we use negative numbers (i.e. draw our parallelogram in a different quadrant)?

8. When the determinant of a matrix is zero, what does the corresponding parallelogram look like? Are rows of the matrix linearly dependent or independent?

9. Draw a shape analogous to a parallelogram in three dimensions. This is called a parallelepiped. How do you find the volume of a parallelepiped?

10. Find the volume of the parallelepiped spanned by $(1, 3, 0), (2, 8, 0), (1, 0, -1)$ using volume-preserving operations analogous to those you discovered for 2X2's. Are the vectors linearly independent?