2016-17 GRADUATE COURSE DESCRIPTIONS

MATH 201 A-B-C (FWS), Akemann/Ponce, *Real Analysis*
Measure theory and integration. Point set topology. Principles of functional analysis. L^p spaces. The Riesz representation theorem. Topics in real and functional analysis.

MATH 202 A-B-C (FWS), Labutin/Putinar, *Complex Analysis*
Analytic functions. Complex integration. Cauchy's theorem. Series and product developments. Entire functions. Conformal mappings. Topics in complex analysis.

MATH 206 A (F), Chandrasekaran, *Matrix Analysis & Computation*
Graduate level-matrix theory with introduction to matrix computations. SVDs, pseudoinverses, variational characterization of eigenvalues, perturbation theory, direct and interative methods for matrix computations.

MATH 206 B (W), Petzold, *Numerical Simulation*
Linear multistep methods and Runge-Kutta methods for ordinary differential equations: stability, order and convergence. Stiffness. Differential algebraic equations. Numerical solution of boundary value problems.

MATH 206 C (S), Ceniceros, *Numerical Solution of Partial Differential Equations - Finite Difference Methods*
Finite difference methods for hyperbolic, parabolic and elliptic PDEs, with application to problems in science and engineering. Convergence, consistency, order and stability of finite difference methods. Dissipation and dispersion. Finite volume methods. Software design and adaptivity.

MATH 206 D (F), Garcia-Cervera, *Numerical Solution of Partial Differential Equations - Finite Element Methods*
Weighted residual and finite element methods for the solution of hyperbolic, parabolic and elliptical partial differential equations, with application to problems in science and engineering. Error estimates. Standard and discontinuous Galerkin methods.

MATH 220 A-B-C (FWS), Jacob/Huisgen-Zimmermann, *Modern Algebra*
Group theory, ring and module theory, field theory, Galois theory, other topics.

MATH 221 A (F), Cooper, *Foundations of Topology*
Metric spaces, topological spaces, continuity, Hausdorff condition, compactness, connectedness, product spaces, quotient spaces. Other topics as time allows.

MATH 221 B (W), Long, *Homotopy Theory*
Homotopy groups, exact sequences, fiber spaces, covering spaces, van Kampen Theorem.

MATH 221 C (S), McCammond, *Differential Topology*
Topological manifolds, differentiable manifolds, transversality, tangent bundles, Borsuk-Ulam theorem, orientation and intersection number, Lefschetz fixed point theorem, vector fields.

MATH 225 A (W), Zhang, *Topics in Number Theory*
Selected topics in Number Theory.

MATH 227 A (F), McCammond, *Advanced Topics in Geometric and Algebraic Topology*
This course will cover the foundations of geometric group theory. Topics include classical combinatorial group theory, metrics on groups, Dehn's fundamental problems, hyperbolic geometry, Gromov hyperbolic groups, ends and boundaries of groups and splittings and quasi-convexity.

MATH 227 B (W), Wang, *Advanced Topics in Geometric and Algebraic Topology*
Quantum invariants of 3-manifolds and the volume conjecture: This will be an introduction to quantum invariants of 3-manifolds and knots. One interesting conjecture is that for hyperbolic knots, the hyperbolic volume of a knot is the same as an asymptotic rate of the colored Jones polynomials of the knot at certain roots of unity. Our approach will be based on state-sum of triangulations: 3-manifolds including knot complements are triangulated and the quantum invariants are sums over colorings of the triangulations such as the Turaev-Viro invariants.

MATH 227 C (S), Bigelow, *Advanced Topics in Geometric and Algebraic Topology*
Planar algebras and their representations [This is tentative, and I am open to suggestions.] Planar algebras are made of diagrams that look like tangles, with operations given by connecting diagrams together. Representations map diagrams to matrices, with operations like matrix multiplication and tensor product. There are applications to knot theory, where a knot diagram maps to a one-by-one matrix, which is a knot invariant. There might be applications to quantum computing, with diagrams describing paths taken by particles, and matrices describing the effect of that computation.

MATH 232 A-B (WS), Cooper/Bigelow, *Algebraic Topology*
Singular homology and cohomology, exact sequences, Hurewicz theorem, Poincare duality.

MATH 236 A-B (FW), Goodearl, *Homological Algebra*
Algebraic construction of homology and cohomology theories, aimed at applications to topology, geometry, groups and rings. Special emphasis on hom and tensor functors; projective, injective and flat modules; exact sequences; chain complexes; derived functors, in particular, ext and tor.

MATH 240 A-B-C (FWS), Zhou/Ye, *Introduction to Differential Geometry and Riemannian Geometry*
Topics include geometry of surfaces, manifolds, differential forms, Lie groups, Riemannian manifolds, Levi-Civita connection and curvature, curvature and topology, Hodge theory. Additional topics such as bundles and characteristic classes, spin structures Dirac operator, comparison theorems in Riemannian geometry.

MATH 241 A-C (F S), Wei, *Topics in Differential Geometry*
Manifolds with integral Ricci Curvature Bounded from Below
Integral Ricci curvature lower bound is much weaker than pointwise bound. Many geometric problems lead to integral curvatures; for example, the isospectral problems, geometric variational problems and extremal metrics, and Chern-Weil's formula for characteristic numbers. Thus, integral curvature bounds can be viewed as an optimal curvature assumption here. We will study the geometry and topology of manifolds with integral curvature bounds, especially the recent development. Another possibility is about Eigenvalues Comparison.

MATH 241 B (W), Dai, *Topics in Differential Geometry*
Index theory and its geometric applications
The Atyiah-Singer index theory is one of the landmark results in 20th century mathematics, unifying several great theorems in differential geometry, algebraic geometry, and differential topology. In this

course we will introduce the Dirac operator, characteristic classes, the Atiyah-Singer index theorem, as well as the Atiyah-Patodi-Singer index theorem. The remaining part will be devoted to its geometric applications, including the stability issues of Einstein metrics, as well as the construction of interesting geometric invariants which can distinguish various geometric structures.

MATH 241 C (S), Wei, *Topics in Differential Geometry*

MATH 246 A-B-C (FWS) Birnir, *Partial Differential Equations*
First-order nonlinear equations; the Cauchy problem, elements of distribution theory an Sobolev spaces; the heat, wave, and Laplace equations; additional topics such as quasilinear symmetric hyperbolic systems, elliptic regularity theory.

MATH 260AA (W), Garcia-Cervera, *Calculus of Variations*
The Calculus Of Variations is concerned with solving extremal problems for a given functional. One of the first problems proposed and solved by Newton is the following: What should be the shape of a surface of revolution that moves in a fluid at constant speed along its axis in order to exert minimal resistance. To solve this problem, one minimizes the functional
_(**y**) =
Z b
a
y(**x**)(**y0**(**x**))3
1 + (**y0**(**x**))2 **dx**
among functions that belong to an appropriate space. As we can see in this example, in a typical problem in the calculus of variations one looks for a function, and not just a real number, as in optimization problems studied in Calculus. Typical questions that one tries to answer are whether a minimizer actually exists (which involves the notion of convexity, weak convergence, and lower semicontinuity of functionals, for example), whether the minimizer is unique, and basic properties of minimizers. One important issue is what happens when minimizers fail to exist in the space under consideration.

By now the Calculus of Variations has developed into a very sophisticated area of Analysis, with deep connections to Convex Analysis, and appears in a large number of areas such as Functional Analysis, Differential Equations, Applied Mathematics, Differential Geometry, etc.

A brief tentative list of contents follows:
1. Fundamentals of Convex Analysis.
2. Duality and Convex Variational Problems.
3. The Direct Method in the Calculus of Variations.
4. Compensated Compactness and Concentration Compactness.
5. Weak Convergence Methods for Nonlinear Partial Differential Equations.
6. Variational Convergence: Homogenization.

Prerequisites: Basic knowledge of ODEs and PDEs (at the level of the Math 214 and 215 series for non-math majors, and the level of Math 243 and 246 for math majors). Basic knowledge of Functional Analysis (at the level of the Math 201 series).

References: Although some of the material will be extracted from published research articles, we will use some of the following references:
1. Introduction to the Calculus of Variations, by Bernard Dacorogna.
2. Variational Methods: Applications to Nonlinear Partial Differential Equations and Hamilto-

nian Systems, by Michael Struwe.
3. Convex Analysis and Variational Problems, by Ivar Ekeland and Roger Temam.
4. Weak Convergence Methods for Nonlinear Partial Differential Equations, by Lawrence Evans.
5. Weak Continuity and Weak Lower Semicontinuity of Non-Linear Functionals, by Bernard Dacorogna.

MATH 260EE (FWS), TBA, *Graduate Student Colloquium*
Topics in algebra, analysis, applied mathematics, combinatorial mathematics, functional analysis, geometry, statistics, topology, by means of lectures and informal conferences with members of faculty.

MATH 260F (F), Atzberger, *Special Topics of Advanced Numerical Analysis*

Stochastic analysis concerns the mathematics of randomness and uncertainty. Examples include the erratic motions of Brownian particles in a fluid due to molecular-level collisions, fluctuations in the stock market, or other types of randomness accounting statistically for unresolved degrees of freedom in a dynamical system. An important class of stochastic processes can be described in a manner analogous to a differential equation. However, unlike deterministic differential equations the coefficients or functions appearing are random and can give rise to solutions that are highly irregular and non-differentiable. This requires new approaches for their analysis building on and extending many of the concepts of calculus and analysis, such as integration and changes of variable, but now in the setting of measure theory on spaces of functions. In this course we introduce the mathematical foundations for analyzing stochastic processes and make connections with other branches of mathematics, such as the analysis of Elliptic partial differential equations and probabilistic methods for existence and uniqueness. The course also covers asymptotic methods for reducing complex models to stochastic descriptions and numerical methods for stochastic differential equations. More details concerning the specific topics can be found below.

· _Introduction
o Historic Motivations.
o Einstein's Theory for Brownian Motion (1905 paper).
o Langevin and Smoluchowski Dynamics.
o Contributions of Ito, McKean, Weiner, Dynkin, Kurtz, and others.

· _Stochastic Processes
o Random Walks: Lattice Process, Poisson Process, Markov-Chain.
o Stopping Times.
o Filtration of Processes.
o Martingales.
o Ito Process.
o Infinitesimal Generator.

· _Stochastic Differential Equations (SDEs)
o Ito Integral.
o Stratonovich Integral.
o Stochastic Differential Equation: Ito vs Stratonovich Type.
o Existence and Uniqueness for SDEs.

· _Stochastic Calculus
o Ito's Formula.

o Ito's Isometry.

o Girsanov Theorem.
o Infinitesimal Generator for SDE.
o Forward and Backward Kolomogorov PDEs.
o Feynman-Kac Formula.
o Martingale Representation Theorem.

· _Stopping Times and Optimal Control
o Dynkin's Theorem.
o Boundary Crossing-Time Problem.
o Boundary Distribution Problem.
o Stopping Time Representation Theorem.
o Stochastic Methods for Analysis of Elliptic PDEs.
o Hamilton-Jacobi-Bellman PDE.
o Stochastic Control.

· _Numerical Methods
o Euler-Marayuma Method and Runge-Kutta Methods.
o Exponential Time-Stepping Methods.
o Weak vs Strong Approximation.
o Monte-Carlo Methods, Quasi-Monte-Carlo Methods, and Variance Reduction.
o Wiener Chaos.
o Karhunen–Loève Theorem.
o Stochastic Finite Difference Methods and Finite Element Methods for SPDEs.

· _Special Topics
o Stochastic Averaging and Multiscale Analysis [8,9].
o Fluctuating Hydrodynamics [8].
o Dynamic Density Functional Theory [10].
o Stochastic Resonance [7].
o Lévy processes.
o Black-Scholes-Merton Financial Pricing Theory.
o Mailliavin Calculus.

MATH 260F (W), Ceniceros, *Special Topics of Advanced Numerical Analysis*
The course will focus on some very useful numerical methods and their beautiful mathematical underpinnings for ordinary and partial differential equations, not covered in our graduate numerical analysis (206) series. The course will depart from the traditional setting in two ways. First, the topics will be presented in the background of open, modern computational problems and challenging multi-scale applications. Second, the student will be assigned a special project during the first week of classes and will do a presentation with his/her results at the end of the course.
Main Topics:
1. Spectral Methods.
2. Multi-resolution.
3. Exponential Time Differencing.
4. Stiff Problems. Linearly Implicit Methods.
5. Implicit Runge-Kutta Methods. Gauss and Lobatto Methods.

6. High order Upwinding. TVD and TVB schemes.

Pre-requisites: Basic Numerical Analysis, Undergraduate Partial and Ordinary Differential Equations.

Bibliography:
The course material will be drawn from the following monographs as well as from several research papers :

1. B. Mercier. An Intoduction of the Numerical Analysis of Spectral Methods. Springer Verlag, 1989.
2. D. Gottlieb and S. Orszag. Numerical Analysis of Spectral Methods. SIAM, 1977.
3. C. Canuto, M. Hussaini, A. Quarteroni, and T. Zhang. Spectral Methods. Fundamentals in Single Domain. Springer, 2006.
4. E. Hairer, S.P. Norsett, G.Wanner. Solving Ordinary Differential Equations I. Nonstiff Problems. Springer Verlag, 1987.
5. E. Hairer and G. Wanner. Solving Ordinary Differential Equations II. Stiff Problems. Springer Verlag, 1996.

MATH 260HH (S), Stopple, *Theory of the Riemann Zeta Function, with Topics in Analysis*
Prerequisites: 202AB (or CCS complex analysis) and a good background in real analysis (at least 118ABC).

This course develops the basic properties of $\zeta(s)$, with applications to the study of prime numbers, including analytic continuation and functional equation, zero free regions, Riemann's explicit formula, distribution of zeros, zeros on the critical line, distribution of values, Lindelof hypothesis, connections to random matrix theory. Along the way I'll introduce analysis topics not necessarily covered in introductory courses: Euler-MacLaurin summation, the Riemann Stieltjes integral, Abelian and Tauberian theorems, Cesaro and Abel summation Jensen's Theorem, Hadamard Three Circle Theorem, Phragmen-Lindelof Principle Fourier Transform, Poisson Summation, Heisenberg Uncertainty.

MATH 260K (S), Yang, *Introduction to Asymptotic and Perturbation Methods*
Often the solution to a given problem presents a variety of spatial and temporal scales. For example boundary layers, high frequency oscillations, or small perturbations from equilibrium states are situations that appear often in a wide range of applications, say fluid mechanics, material sciences and geophysics. The objective of this course is to familiarize ourselves with perturbation methods and asymptotic analysis, with special emphasis on the study of both ordinary and partial differential equations. The examples treated in the course will cover a wide range of disciplines within Applied Mathematics, and have been chosen to illustrate situations that appear often in certain areas of research.

Contents
1. Introduction.
1.1 Order symbols.
1.2 Asymptotic expansion of functions: Asymptotic series.
1.3 Asymptotic expansion of integrals:
1.3.1 Watson's lemma.
1.3.2 Laplace's approximation.
1.3.3 The method of stationary phase.
1.4 Regular expansions for ODEs and PDEs.
2. Singular perturbations for ODEs.
2.1 Problems with interior transition layers.
2.2 Problems with boundary layers.

2.3 Problems with multiple transition layers.

3. Limit procedures in Partial Differential Equations and the Calculus of Variations.
3.1 Ginzburg-Landau equations: **_2_u** + (1 □ j **uj2)u** = 0.
3.2 Water waves.
3.3 Instabilities in fluids.
3.4 Burger's equation for viscous fluids.
3.5 Nonlinear Shrodinger equation and Since-Gordon equations.
4. The method of multiple scales for ODEs.
4.1 Forced motion: Resonance.
4.2 Finite difference equations.
5. The method of multiple scales in PDEs.
5.1 Second order equations.
5.2 Nonlinear wave propagation.
5.3 Homogenization.
6. Variational Convergence and □ -limit

References

[1] H. Attouch. Variational Convergence for Functions and Operators**. Pitman Advanced Publishing Program, Boston-London-Melbourne, 1984.**
[2] N. Bleistein and R. A. Handelsman. Asymptotic Expansions of Integrals**. Dover, New York, 1975.**
[3] M. H. Holmes. Introduction to Perturbation Methods**. Texts in Applied Mathematics, Volume 20. Springer-Verlag, Berlin-New York, 1994.**
[4] J. Kevorkian and J.D. Cole. Multiple Scales and Singular Perturbation Methods**. Applied Mathematical Sciences, Volume 114. Springer-Verlag, Berlin-New York, 1995.**
[5] Gianni Dal Maso. Introduction to □-Convergence. **Progress in Nonlinear Differential Equations and their Applications. Birkhauser, Boston, MA, 1993.**
[6] Samuel S. Shen. A Course on Nonlinear Waves**. Kluwer Academic Publishers, 1993.**

MATH 260L (F) Sideris, *Partial Differential Equations in Elasto- and Hydro-dynamics*
Provide a kinematic framework to describe the motion of fluids and solids. Derive the fundamental equations of motion using Newton's laws. Explore constitutive assumptions for internal material forces: Cauchy elastic materials, visco-elastic materials, isotropic materials, compressible and incompressible fluids, Newtonian and non-Newtonian fluids, etc. Variational formulations of conservative models. Analysis of elementary wave interactions. A survey of analytical results on the local and global well-posedness for the equations of motion in elasto- and hydro-dynamics. Detailed analysis of one or two examples, time permitting.

Prerequisites: Math 246ABC. Solid grounding in undergraduate linear algebra and multivariable analysis. Basic understanding of mechanics.

This will be a seminar-style course, and it could not be used to fulfill area requirements. Students will be asked to select the P/NP grading option.

MATH 260Q, (FS), Huisgen-Zimmermann, *Algebraic groups, invariant theory, and applications to representations of algebras*

We will start with a fairly extensive survey of basic algebraic geometry to provide the fundamentals needed in the sequel. (The introduction to algebraic geometry given by D. Morrison this year will not be a prerequisite.)

The primary focus of the course is the study of linear algebraic groups (i.e., groups carrying an additional structure -- that of an affine variety -- such that the group operations are morphisms of varieties). We will first explore the intrinsic algebraic and geometric structure of such groups and then discuss their actions on both affine and projective varieties. The results will lend themselves to a first installment of applications to representations of algebras. In particular, we will prove Gabriel's theorem concerning the representation types of quivers.

Given a suitable action of an algebraic group G on a variety V, the ring of G-invariant regular functions from V to the base field is of pivotal importance (a function is G-invariant if it is constant on the orbits); we will prove classical results about the structure of the ring of invariants. In favorable cases, the G-invariant functions permit us to factor the G-action out of the variety V, to the effect that the set of G-orbits in turn carries a geometry; the resulting "geometric quotient" displays information about the original setup in distilled form. Again, we will give applications to the representation theory of finite dimensional algebras.

Prerequisites: 220 ABC or consent of the instructor. For the applications, it will be useful to have a modest background in homological algebra (e.g., conveyed in 236A); for those lacking such a background, summaries without proofs will be given.

MATH 501 (F), Ograin, *Teaching Assistant Training*
Consideration of ideas about the process of learning mathematics and discussion of approaches to teaching.