2017-18 GRADUATE COURSE DESCRIPTIONS

MATH 201 A-B-C (FWS), Labutin/Akemann, *Real Analysis*
Measure theory and integration. Point set topology. Principles of functional analysis. L^p spaces. The Riesz representation theorem. Topics in real and functional analysis.

MATH 206 A (F), Chandrasekaran, *Matrix Analysis & Computation*
Graduate level-matrix theory with introduction to matrix computations. SVDs, pseudoinverses, variational characterization of eigenvalues, perturbation theory, direct and interative methods for matrix computations.

MATH 206 B (W), Petzold, *Numerical Simulation*
Linear multistep methods and Runge-Kutta methods for ordinary differential equations: stability, order and convergence. Stiffness. Differential algebraic equations. Numerical solution of boundary value problems.

MATH 206 C (S), Yang, *Numerical Solution of Partial Differential Equations - Finite Difference Methods*
Finite difference methods for hyperbolic, parabolic and elliptic PDEs, with application to problems in science and engineering. Convergence, consistency, order and stability of finite difference methods. Dissipation and dispersion. Finite volume methods. Software design and adaptivity.

MATH 206 D (F), Garcia-Cervera, *Numerical Solution of Partial Differential Equations - Finite Element Methods*
Weighted residual and finite element methods for the solution of hyperbolic, parabolic and elliptical partial differential equations, with application to problems in science and engineering. Error estimates. Standard and discontinuous Galerkin methods.

MATH 209 (S), Cooper, *Introduction to Mathematical Logic*
- Ordinals, set theory, Godel's Constructible Universe, Consistency of Axiom of Choice.
- Turing machines, Theorem: Computable Functions = Recursive Functions, example of non-computable function.
- Propositional calculus, predicate calculus, first-order logic, completeness, logical compactness, undecidability.

If there is time, and sufficient interest, additional topics might include applications to non-standard analysis: infinitesimals, real algebraic geometry.

Preparatory reading:
What is Mathematical Logic; Crossley, et al.
A Tour Through Mathematical Logic; Robert Wolf

MATH 220 A-B-C (FWS), Agboola, *Modern Algebra*
Group theory, ring and module theory, field theory, Galois theory, other topics.

MATH 221 A (F), Bigelow, *Foundations of Topology*
Metric spaces, topological spaces, continuity, Hausdorff condition, compactness, connectedness, product spaces, quotient spaces. Other topics as time allows.

MATH 221 B (W), Cooper, *Homotopy Theory*
Homotopy groups, exact sequences, fiber spaces, covering spaces, van Kampen Theorem.

MATH 221 C (S), Long, *Differential Topology*
Topological manifolds, differentiable manifolds, transversality, tangent bundles, Borsuk-Ulam theorem, orientation and intersection number, Lefschetz fixed point theorem, vector fields.

MATH 225 A-B-C (FWS), Zhang, *Topics in Number Theory*
The course will cover the basic concepts and techniques of number theory.

In Math 225A, we start with elementary number theory, but also introduce some analytic and algebraic tools; in Math 225B, we focus on the algebraic and analytic aspects that will be continued to Math 225C; in Math 225C, we may bring some problems in current researches to discuss, and lead the graduate students to their own research.

MATH 227 A (F), McCammond, *Advanced Topics in Geometric and Algebraic Topology*
The geometry of Coxeter groups. The discrete groups generated by reflections that act on nicely spheres and euclidean spaces include all of the isometry groups of the regular polytopes (i.e. regular m-gons, the Platonic solids and their higher dimensional generalizations) as well as the crystallographic groups that are at the heart of the theory of Lie Groups and Lie Algebras. The general class of Coxeter groups is a vast generalization of these groups. The course will focus on the many remarkable geometric and algebraic properties of this class of groups.

MATH 227 B (W), Wang, *Advanced Topics in Geometric and Algebraic Topology*
Motion groups and their representations. The braid groups and mapping class groups of surfaces are examples of motion groups of a sub-manifold L inside an ambient manifold Y. After a general introduction, we will focus on the motion groups of links=L in the 3-sphere=Y, especially the loop braid groups of the unlinks, and the necklace groups. TQFTs provide representations of the motion groups, but they are not computed in general. Very few explicit representations of motion groups are known and studied mathematically. Motion groups of links in 3-sphere are statistics of string excitations in physics.

MATH 231 A-B (FW), Goodearl, *Lie Algebras and Lie Groups*
This will be an introduction to the subject, the only prerequisites being some background in linear algebra and the 220ABC sequence. The material to be covered includes the general theory of Lie algebras, structure and classification of semisimple Lie algebras, representation theory of Lie algebras, and an introduction to Lie groups and their Lie algebras. More specific topics include nilpotent, solvable, and simple / semisimple Lie algebras; Engel's, Lie's and Weyl's Theorems; root systems; Weyl groups; Dynkin diagrams; enveloping algebras; weight spaces of representations.

The course will be based on the texts:

J.E. Humphreys, Introduction to Lie Algebras and Representation Theory, Grad. Texts in Math.

Springer 1972 (3rd revised printing, 1982), and

J.-P. Serre, Complex Semisimple Lie Algebras, Springer 1987 (reprinted 2001).

MATH 232 A-B (WS), McCammond/Bigelow, *Algebraic Topology*
Singular homology and cohomology, exact sequences, Hurewicz theorem, Poincare duality.

MATH 240 A-B-C (FWS), Dai/Ye/Wei, *Introduction to Differential Geometry and Riemannian Geometry*
Topics include geometry of surfaces, manifolds, differential forms, Lie groups, Riemannian manifolds, Levi-Civita connection and curvature, curvature and topology, Hodge theory. Additional topics such as bundles and characteristic classes, spin structures Dirac operator, comparison theorems in Riemannian geometry.

MATH 241 A (F), Zhou, *Topics in Differential Geometry*
Introduction to minimal surfaces
The theory of minimal surfaces is an important topic in many subjects, including geometry, topology, analysis, and physics. In this course, we will give a brief introduction to this beautiful theory. We will start with the basic first and second variation formulae, monotonicity formula, and maximum principle. Then we will cover several classical curvature estimates. Finally, we will choose between the following more advanced topics: the classical Plateau problem, Choi-Schoen compactness theorem, or the Positive Mass Theorem in General Relativity.

MATH 241 B (W), Dai, *Topics in Differential Geometry*

MATH 241 C (S), Ye, *Topics in Differential Geometry*

MATH 243 A-B-C (FWS) Ponce/Sideris, *Ordinary Differential Equations*
Existence and stability of solutions, Floquet theory, Poincare-Bendixson theorem, invariant manifolds, existence and stability of periodic solutions, Bifurcation theory and normal forms, hyperbolic structure and chaos, Feigenbaum period-doubling cascade, Ruelle-Takens cascade.

MATH 260EE (FWS), TBA, *Graduate Student Colloquium*
Topics in algebra, analysis, applied mathematics, combinatorial mathematics, functional analysis, geometry, statistics, topology, by means of lectures and informal conferences with members of faculty.

MATH 260J (F) Atzberger*, Machine Learning: Foundations and Applications*
This special topics course will survey current approaches in machine learning, their mathematical foundations, and practical computational methods. Recent advances in measurement and scientific simulation are resulting in a plethora of data. This presents a number of challenges both for the design of high-throughput experiments/simulations and for the interpretation of the resulting large sets of data. Advances in combining approaches, such as Bayesian statistics, with large-scale scientific computation are resulted in new methodologies and algorithms for inferring information from such data. In this

special topics class, we will give a survey of current Machine Learning techniques and practical methods that draw on results from stochastic analysis. The beginning introductory materials of the course will use the books *"The Elements of Statistical Learning: Data Mining, Inference, and Prediction"* by Hastie, Tibshirani, Friedman and *"Stochastic Differential Equations: An Introduction with Applications,"* by Bernt Øksendal. The remaining part of the course will be based on special lecture materials and recent papers in the literature.

More details concerning the specific topics can be found below.
Sample of Topics:

Introduction

o Historic Motivations.
o Bayesian vs classical statistics.
o Maximum likelihood methods.
o Uncertainty quantification.
o Markov-chain Monte-Carlo sampling.
o Motivating applications from the sciences, engineering, and finance.

Introduction to Stochastic Processes

o Random Walks: Lattice Process, Poisson Process, Markov-Chain.
o Brownian Motion, Martingales, Ito Process.
o Sigma-Algebras, Filtration of Processes, Conditional Expectation.
o Stochastic Differential Equations (SDEs), Ito Integral, Ito's Formula.
o Stochastic Finite Difference Methods and Finite Element Methods for SPDEs.
o Karhunen–Loève Theorem and Expansions.
o Wiener Chaos Expansion.
o SDEs connection to PDEs, Infinitesimal Generator, Forward and Backward Kolomogorov PDEs.
o Stochastic Methods for Analysis of Elliptic and Parabolic PDEs.

Statistical Inference and Machine Learning

o Supervised Learning Methods
o Linear Methods for Regression and Classification
o Kernel smoothing methods
o Parametric vs non-parametric regression
o Model selection and bias-variance trade-offs
o Neural Networks, Support Vector Machines, Graphical Models.

Numerical Methods for Machine Learning

o Sparse Matrix Methods
o Preconditioners and Iterative Methods
o Conjugate Gradient Methods
o Non-linear Optimization Methods
o Markov-Chain Monte-Carlo Sampling
o Sampling with stochastic processes
o Dimension reduction (graph Laplacian, eigen-analysis)
o Stochastic averaging and multiscale analysis.

References:
1. *The Elements of Statistical Learning Data Mining, Inference, and Prediction,* T. Hastie, R. Tibshirani, J. Friedman, (2013).
2. *Machine Learning: A Probabilistic Perspective*, K. P. Murphy, (2012).
3. *Stochastic Differential Equations: An Introduction with Applications,* Oksendal, B. K., (2003).
4. *Handbook of Stochastic Methods: for Physics, Chemistry and the Natural Sciences,* Gardiner, C., (2009).

MATH 260K (S), Yang, *Introduction to Asymptotic and Perturbation Methods*
Often the solution to a given problem presents a variety of spatial and temporal scales. For example boundary layers, high frequency oscillations, or small perturbations from equilibrium states are situations that appear often in a wide range of applications, say fluid mechanics, material sciences and geophysics. The objective of this course is to familiarize ourselves with perturbation methods and asymptotic analysis, with special emphasis on the study of both ordinary and partial differential equations. The examples treated in the course will cover a wide range of disciplines within Applied Mathematics, and have been chosen to illustrate situations that appear often in certain areas of research.

Contents
1. Introduction.
1.1 Order symbols.
1.2 Asymptotic expansion of functions: Asymptotic series.
1.3 Asymptotic expansion of integrals:
1.3.1 Watson's lemma.
1.3.2 Laplace's approximation.
1.3.3 The method of stationary phase.
1.4 Regular expansions for ODEs and PDEs.

2. Singular perturbations for ODEs.
2.1 Problems with interior transition layers.
2.2 Problems with boundary layers.
2.3 Problems with multiple transition layers.

3. Limit procedures in Partial Differential Equations and the Calculus of Variations.
3.1 Ginzburg-Landau equations: **_2_u** + (1 j**u**j**2**)**u** = 0.
3.2 Water waves.
3.3 Instabilities in fluids.
3.4 Burger's equation for viscous fluids.
3.5 Nonlinear Shrodinger equation and Since-Gordon equations.

4. The method of multiple scales for ODEs.
4.1 Forced motion: Resonance.
4.2 Finite difference equations.

5. The method of multiple scales in PDEs.
5.1 Second order equations.
5.2 Nonlinear wave propagation.

5.3 Homogenization.

References:
[1] N. Bleistein and R. A. Handelsman**,** *Asymptotic Expansions of Integrals* Dover, New York, 1975.
[2] W. E., *Principles of Multiscale Modeling,* Cambridge University Press, UK, 2011.
[3] M. H. Holmes, *Introduction to Perturbation Methods*, Texts in Applied Mathematics, Volume 20. Springer-Verlag, Berlin-New York, 1994.
[4] J. Kevorkian and J.D. Cole, *Multiple Scales and Singular Perturbation Methods*, Applied Mathematical Sciences, Volume 114. Springer-Verlag, Berlin-New York, 1995.
[5] Gianni Dal Maso, *Introduction to -Convergence: Progress in Nonlinear Differential Equations and their Applications,* Birkhauser, Boston, MA, 1993.
[6] Samuel S. Shen, *A Course on Nonlinear Waves*, Kluwer Academic Publishers, 1993.

MATH 260L (F), Birnir,
Turbulence, Theory, Experiments and Numerical Simulations
The modern theory of turbulence will be explored. We will develop the theory of stochastic closure in turbulence and show how it predicts experimental results in homogeneous turbulence and turbulent boundary value problems. Applications to wind tunnels and wind farms will be explored. Then we will develop the applications of the theory to numerical simulations both RANS and LES. The textbook that will be used is
“The Kolmogorov-°©-Obukhov Theory of Turbulence” by Björn Birnir, Springer Verlag.

If there is time, developments in Lagrangian turbulence will also be covered.

MATH 260Q (FW), Huisgen-Zimmermann, ***Representation Theory of Groups and Algebras***
We will start by assembling an arsenal of interesting examples of finite dimensional algebras and introducing the basic tools for tackling them: categories, functors, chain conditions, radical, decomposition theory.

Then we will focus on the representation theory of finite groups (= the representation theory of the corresponding group algebras). The first major goal is to develop the classical character theory in characteristic zero; it amounts to the time-honored approach to understanding mathematical objects by studying telltale functions with values in a base field. The purely algebraic viewpoint will be supplemented by Young's combinatorial approach to characters by way of ``Young tableaux"; in particular, they lead to full mastery of the representation theory of symmetric groups.

In a second part, we will move to more general algebras. The ultimate goal is to understand the basic building blocks of their representations and classify them, a realistic objective only if the considered algebra has "finite (or tame) representation type", meaning that the number of such building blocks is finite (or controlled infinite). The trip will conclude with the conjectures of Brauer and Thrall proposing characterizations of algebras of finite representation type. These conjectures were confirmed, first by Roiter, then by Auslander in a broader setting. The finessed strategy which we will be adopting will expose us to the "modern" methods of the subject.

MATH 260SS (W), Ceniceros, ***Mathematical and Computational Topics of Data Science***

This is a seminar style course on some mathematical and computationals aspects of Data Science. The emphasis will be on high dimensional problems. We intend to cover some or all of the topics below:

1. Principal component analysis and SVD computation.
2. Introduction to Random Matrix Theory.
3. Elements of Graph Theory.
4. Graph Laplacian. Diffusion Maps.
5. Multi-resolution analysis on graphs.
6. Random walks on graphs. Perron theorem. PageRank.
7. Mean field games.

Prerequisites: Basic Probability, Linear Algebra, Analysis, and introductory Numerical Analysis.
Grading: Class participation 100%
Bibliography: The course materials will be drawn from research articles and from some textbookds relevant to each topic. Reading materials will be provided for each topic.

MATH 501 (F), Garfield, *Teaching Assistant Training*
Consideration of ideas about the process of learning mathematics and discussion of approaches to teaching.