2014-15 GRADUATE COURSE DESCRIPTIONS

MATH 201 A-B-C (FWS), Akemann, *Real Analysis*

MATH 202 A-B-C (FWS), Putinar, *Complex Analysis*

MATH 206 A (F), Chandrasekaran, *Matrix Analysis & Computation*
Graduate level-matrix theory with introduction to matrix computations. SVDs, pseudoinverses, variational characterization of eigenvalues, perturbation theory, direct and iterative methods for matrix computations.

MATH 206 B (W), Petzold, *Numerical Simulation*

MATH 206 C (S), Ceniceros, *Numerical Solution of Partial Differential Equations - Finite Difference Methods*

MATH 206 D (F), Atzberger, *Numerical Solution of Partial Differential Equations - Finite Element Methods*

MATH 209 (F), Cooper, *Set Theory/Introduction to Mathematical Logic*
Propositional calculus, predicate calculus, the tableau method, first-order logic, completeness, logical compactness, computable functions, undecidability, set theory. If there is time, and sufficient interest, additional topics might include: applications to non-standard analysis, infinitesimals, real algebraic geometry.

Preparatory reading:
What is Mathematical Logic; Crossley, et al.
A tour through mathematical logic; Robert Wolf

MATH 220 A-B-C (FWS), Agboola, *Modern Algebra*
Group theory, ring and module theory, field theory, Galois theory, other topics.

MATH 221 A (F), Bigelow, *Foundations of Topology*
Metric spaces, topological spaces, continuity, Hausdorff condition, compactness, connectedness, product spaces, quotient spaces. Other topics as time allows.
MATH 221 B (W), Millett, *Homotopy Theory*
Homotopy groups, exact sequences, fiber spaces, covering spaces, van Kampen Theorem.

MATH 221 C (S), Long, *Differential Topology*
Topological manifolds, differentiable manifolds, transversality, tangent bundles, Borsuk-Ulam theorem, orientation and intersection number, Lefschetz fixed point theorem, vector fields.

MATH 227 A (F), Long, *Advanced Topics in Geometric and Algebraic Topology*
*Algebra and low dimensional topology:* The course will explore interactions between various applications of algebra to topics in low-dimensional topology. In part this may be guided by the interests of the audience.

MATH 227 B (W), McCammond, *Advanced Topics in Geometric and Algebraic Topology*
*Exceptional mathematics:* Many parts of mathematics have classification theorems which list infinite families of examples and a few additional exceptional objects. From one field to another, the infinite families are often related to the other infinite families (and I'll call that classical mathematics) and the exceptions are often related to the exceptions (and this is exceptional mathematics). The topics touched on will range from polytopes and reflections groups to symmetric spaces and Lie theory, from abstract geometries to sphere packings to high-dimensional hyperbolic orbifolds and sporadic finite simple groups.

MATH 227 C (S), Scharlemann, *Advanced Topics in Geometric and Algebraic Topology*
A topic of current interest in low-dimensional topology, yet to be decided. Here 'low-dimensional' means either 3-manifolds or 4-manifolds. Or, if developments allow, the in-between world of $(3 + 1)$-dimensional manifolds. See, for example [http://lamington.wordpress.com/2013/10/18/scharlemann-on-schoenflies/](http://lamington.wordpress.com/2013/10/18/scharlemann-on-schoenflies/)

MATH 232 A (W), Scharlemann, *Algebraic Topology*
Singular homology and cohomology, exact sequences, Hurewicz theorem, Poincare duality.

MATH 236 A-B (WS), Huisgen-Zimmermann, *Homological Algebra*
Algebraic construction of homology and cohomology theories, aimed at applications to topology, geometry, groups and rings. Special emphasis on hom and tensor functors; projective, injective and flat modules; exact sequences; chain complexes; derived functors, in particular, ext and tor.

MATH 240 A-B-C (FWS), Moore/Ye/Dai, *Introduction to Differential Geometry and Riemannian Geometry*
Topics include geometry of surfaces, manifolds, differential forms, Lie groups, Riemannian manifolds, Levi-Civita connection and curvature, curvature and topology, Hodge theory. Additional topics such as bundles and characteristic classes, spin structures Dirac operator, comparison theorems in Riemannian geometry.

MATH 241 A (F), Wei, *Topics in Differential Geometry*
*Manifolds with integral Ricci Curvature Bounded from Below:* Integral Ricci curvature lower bound is much weaker than pointwise bound. Many geometric problems lead to integral curvatures; for example, the isospectral problems, geometric variational problems and extremal metrics, and Chern-Weil's formula for characteristic numbers. Thus, integral curvature bounds can be viewed as an optimal curvature assumption here. We will study the geometry and topology of manifolds with integral curvature bounds.
MATH 241 B (W), Dai, *Topics in Differential Geometry*

*Geometry and Analysis on Manifolds with lower Ricci Curvature Bound:* There has been tremendous development in the study of manifolds with Ricci curvature bounded from below. The most fascinating aspect here is the interaction of geometry, analysis and topology. We will discuss the most fundamental results in this area including the Cheeger-Gromoll's Splitting Theorem, the Gradient Estimate, First Eigenvalue and Heat Kernel Comparison, Dirichlet and Neumann Eigenvalue Comparison, Heat Kernel Comparison, Isoperimetric Inequality.

MATH 241 C (S), Ye, *Topics in Differential Geometry*

*Introduction to the Ricci Flow:* In this course we'll present the basics of the theory of the Ricci flow. The following is a tentative list of topics to be covered:

1. Short time existence of the Ricci flow on compact manifolds.
2. Perelman's functionals and noncollapsing of the Ricci flow.
3. The logarithmic Sobolev and Sobolev inequalities along the Ricci flow.
5. An introduction to the Yamabe flow.
6. The Ricci flow for manifolds of positive curvature.

MATH 246 A-B-C (FWS) Ponce/Yang/Labutin, *Partial Differential Equations*

First-order nonlinear equations; the Cauchy problem, elements of distribution theory an Sobolev spaces; the heat, wave, and Laplace equations; additional topics such as quasilinear symmetric hyperbolic systems, elliptic regularity theory.

MATH 260A (F), Z. Wang, *Topological Quantum Computation*

The course is an introduction to the mathematical foundations of topological quantum computation. Although it is the second installment of a three-course sequence, it will be largely independent of the first course. While the focus of last course is on the topological side of the field centering on TQFTs, this quarter the course will focus on the algebraic side---the theory of modular categories. The plan is to start with an elementary definition of modular category and introduce the basic techniques for their classification. We will give an introduction to their connections with topological phases of matter and quantum computing.

MATH 260AA (S), Garcia-Cervera, *Calculus of Variations*

The Calculus Of Variations is concerned with solving extremal problems for a given functional. One of the rst problems proposed and solved by Newton is the following: What should be the shape of a surface of revolution that moves in a uid at constant speed along its axis in order to exert minimal resistance. To solve this problem, one minimizes the functional

\[(y) = \int_{a}^{b} \left[ y(x)(y'(x))^3 \right] dx \]

among functions that belong to an appropriate space. As we can see in this example, in a typical problem in the calculus of variations one looks for a function, and not just a real number, as in optimization problems studied in Calculus. Typical questions that one tries to answer are whether a minimizer actually exists (which involves the notion of convexity, weak convergence, and lower semicontinuity of functionals, for example), whether the minimizer is unique, and basic properties of minimizers. One important issue is what happens when minimizers fail to exist in the space.
By now the Calculus of Variations has developed into a very sophisticated area of Analysis, with deep connections to Convex Analysis, and appears in a large number of areas such as Functional Analysis, Differential Equations, Applied Mathematics, Differential Geometry, etc.

A brief tentative list of contents follows:
1. Fundamentals of Convex Analysis.
2. Duality and Convex Variational Problems.
3. The Direct Method in the Calculus of Variations.

Prerequisites: Basic knowledge of ODEs and PDEs (at the level of the Mat 214 and 215 series for non-math majors, and the level of Mat 243 and 246 for math majors). Basic knowledge of Functional Analysis (at the level of the Mat 201 series).

References: Although some of the material will be extracted from published research articles, we will use some of the following references:
1. Introduction to the Calculus of Variations, by Bernard Dacorogna.
5. Weak Continuity and Weak Lower Semicontinuity of Non-Linear Functionals, by Bernard Dacorogna.

MATH 260EE (FWS), TBA, Graduate Student Colloquium
Topics in algebra, analysis, applied mathematics, combinatorial mathematics, functional analysis, geometry, statistics, topology, by means of lectures and informal conferences with members of faculty.

MATH 260K (S), Yang, Introduction to Asymptotic and Perturbation Methods
Often the solution to a given problem presents a variety of spatial and temporal scales. For example boundary layers, high frequency oscillations, or small perturbations from equilibrium states are situations that appear often in a wide range of applications, say fluid mechanics, material sciences and geophysics. The objective of this course is to familiarize ourselves with perturbation methods and asymptotic analysis, with special emphasis on the study of both ordinary and partial differential equations. The examples treated in the course will cover a wide range of disciplines within Applied Mathematics, and have been chosen to illustrate situations that appear often in certain areas of research.

Contents
1. Introduction.
   1.1 Order symbols.
   1.2 Asymptotic expansion of functions: Asymptotic series.
   1.3 Asymptotic expansion of integrals:
      1.3.1 Watson's lemma.
      1.3.2 Laplace's approximation.
      1.3.3 The method of stationary phase.
   1.4 Regular expansions for ODEs and PDEs.
2. Singular perturbations for ODEs.
2.1 Problems with interior transition layers.
2.2 Problems with boundary layers.
2.3 Problems with multiple transition layers.

3.1 Ginzburg-Landau equations: \( _2 u + (1 + j)u) = 0. \)
3.2 Water waves.
3.3 Instabilities in fluids.
3.4 Burger's equation for viscous fluids.
3.5 Nonlinear Shr Æ dinger equation and Since-Gordon equations.

4. The method of multiple scales for ODEs.
4.1 Forced motion: Resonance.
4.2 Finite difference equations.

5. The method of multiple scales in PDEs.
5.1 Second order equations.
5.2 Nonlinear wave propagation.
5.3 Homogenization.

6. Variational Convergence and -limit

References

MATH 260MM, (S) Long/Ballas, Casson's invariant and beyond
The course will develop from scratch an elegant and surprising construction, due to Andrew Casson, of an integer invariant of homology spheres. This turns out to have many powerful applications, for example it can be used to show that there are topological 4-manifolds which are not homeomorphic to any simplicial complex. This invariant was subsequently the basis for an enormous amount of mathematics, including Floer homology. We will meander around this subject and contiguous areas as interest and taste dictates…..

MATH 260P (F) Morrison/Kennard, Bundles, curvature, and anomalies
The course will focus on aspects of curvature which find applications in geometry, topology, and theoretical physics. We will study two kinds of curvature: curvature of Riemannian metrics, and curvature of connections on bundles. Some previous acquaintance with manifolds, Riemannian metrics, and Lie groups will be useful, but these topics will be reviewed. New topics include the Gauss-Bonnet theorem as reformulated by Chern, the Chern-Weil homomorphism, curvature of connections on principal bundles, and the computation of the "anomalies" which sometimes obstruct the existence of a quantum-mechanical physical theory formed from quantum analogues of metrics, curvatures, and differential forms.
MATH 260Q, (W), Goodearl, *Quantum Groups*
This course will offer an introduction to some of the core ideas in a subject called, somewhat mysteriously, "Quantum Groups", a field which arose from mathematical physics in the 1980s and has since developed many connections with areas as diverse as representation theory, noncommutative geometry, and knot theory. Its historical origin was the study of the "quantum Yang-Baxter equation" in quantum statistical mechanics, solutions to which came from the representation theory of "quantized enveloping algebras" of Lie algebras and led to "quantized algebras of functions" on Lie groups.

The prerequisite is just 220ABC or equivalent; background in Lie theory, representation theory, or algebraic geometry is not assumed, but will be developed as needed, along with some relevant noncommutative algebra. A sample list of topics to be discussed follows.

-- Affine algebraic varieties, polynomial function algebras, and their quantizations

-- Lie algebras, enveloping algebras, and their quantizations

-- Noncommutative noetherian rings and rings of fractions

-- Skew polynomial rings and presentations of quantized algebras

-- Hopf algebras

-- Hopf duality between (quantized) algebras of functions on Lie groups and (quantized) enveloping algebras of their Lie algebras

MATH 260Q (S), Morrison, *Calabi-Yau Manifolds*
Calabi--Yau manifolds are defined by a condition in differential geometry, but thanks to Yau's proof of the Calabi conjecture, are largely studied by methods of algebraic geometry. In this course, we will learn the basics of Kähler geometry and special holonomy so that we can understand the differential geometry characterization. Yau's theorem will then be used to translate the condition into algebraic geometry, where we will study Calabi--Yau manifolds anew. We will explore K3 surfaces (a.k.a. Calabi--Yau manifolds in complex dimension 2) in detail, and then learn about construction techniques and properties of Calabi--Yau manifolds in higher dimension.

Although the applications to physics will motivate our choice of topics, no physics background is required for the course.

MATH 501 (F), Ograin, *Teaching Assistant Training*
Consideration of ideas about the process of learning mathematics and discussion of approaches to teaching.